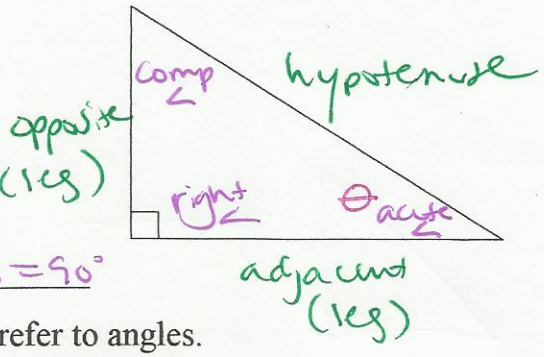
Triangle Trigonometry – Trigonometric Ratios in Right Triangles

A right triangle consists of two important features:

1.) Angles

- right ∠ which is equal to 90°
- acute ∠ which less than 90°
- complementary ∠ which is added to the acute ∠ to = 90°



* Note → The symbol θ (called theta) is used to refer to angles.

2.) Sides

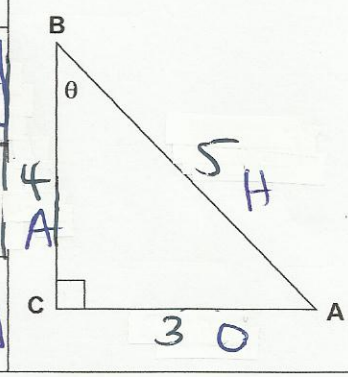
H - hypotenuse which is the longest side and across from right ∠

- legs which are the sides that form right ∠.

A - adjacent side which is the leg that is directly next to the indicated angle (θ)

O - opposite side which is the leg that is directly across from the indicated angle (θ)

Words	Symbol	Definition
Sine	$\sin\theta$	$\sin\theta = \frac{O}{H} \rightarrow \boxed{\sin\theta = \frac{3}{5}}$
Cosine	$\cos\theta$	$\cos\theta = \frac{A}{H} \rightarrow \boxed{\cos\theta = \frac{4}{5}}$
tangent	$\tan\theta$	$\tan\theta = \frac{O}{A} \rightarrow \boxed{\tan\theta = \frac{3}{4}}$



- To find a missing side of a right triangle (given two sides) → must use Pyth. theorem ($a^2+b^2=c^2$)

Note: You WILL have to rationalize the denominator for problems like Example 2.
Remember – Rationalizing the denominator is a way to "simplify" by having the denominator an exact root to produce a rational number (hence the name of the process)

Example 1: Let's review on how to "Rationalize the Denominator".

a.) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{4\sqrt{5}}{\sqrt{25}} = \boxed{\frac{4\sqrt{5}}{5}}$

b.) $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{6\sqrt{3}}{\sqrt{9}}$

$= \frac{{}^{2}\cancel{6}\sqrt{3}}{\cancel{3}_{1}} = \boxed{2\sqrt{3}}$

c.) $\frac{2}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{4 \cdot 2} = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{8}_{4}}$

$= \boxed{\frac{\sqrt{2}}{4}}$

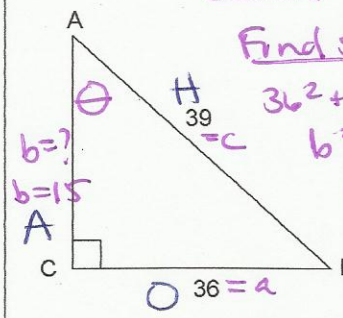
d.) $\frac{48}{3\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{48\sqrt{6}}{3 \cdot 6} = \frac{{}^{8}\cancel{48}\sqrt{6}}{\cancel{18}_{3}}$

$= \boxed{\frac{8\sqrt{6}}{3}}$

Example 2: **Complete each problem (and its parts) below.**

a.) Find the three trigonometric ratios for angle A. → θ = A



Find side b first..

$36^2 + b^2 = 39^2$

$b^2 = 225$

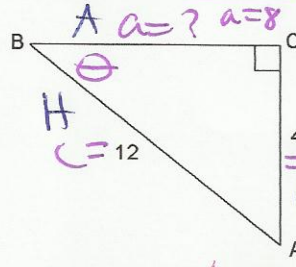
$b = 15$

$\sin\theta = \frac{36}{39} = \frac{12}{13}$

$\cos\theta = \frac{15}{39} = \frac{5}{13}$

$\tan\theta = \frac{36}{15} = \frac{12}{5}$

b.) Find the three trigonometric ratios for angle B. → θ = B



Find side a first..

$a^2 + (4\sqrt{5})^2 = 12^2$

$a^2 + 80 = 144$

$a^2 = 64$

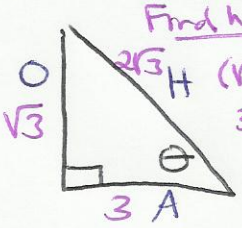
$a = 8$

$\sin\theta = \frac{4\sqrt{5}}{12_3} = \frac{\sqrt{5}}{3}$

$\cos\theta = \frac{8}{12} = \frac{2}{3}$

$\tan\theta = \frac{4\sqrt{5}}{8_2} = \frac{\sqrt{5}}{2}$

c.) Given $\tan\theta = \frac{\sqrt{3}}{3}$, find the other two trig ratios.



Find hyp first..

$(\sqrt{3})^2 + 3^2 = c^2$

$3 + 9 = c^2$

$12 = c^2$

$c = 2\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

$\cos\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6_2} = \frac{\sqrt{3}}{2}$

Things to Know When Solving Triangles → MODE is in DEGREES (check please!)

1.) Capital letters or θ (theta) will always represent angles (Triangle = 180°)

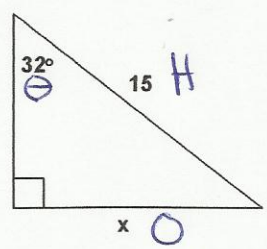
2.) Lowercase letters will always represent sides

3.) a.) If "x" is on top, then multiply #'s b.) If "x" is on bottom, then divide #'s (R÷by L)

4.) Round ALL sides to the nearest tenth! NO RADICALS when asked to SOLVE a triangle!!!!

Example 3: **Find the value of x for a.) , b.) , and c.) but SOLVE triangle ABC for d.) and e.)**

a.)

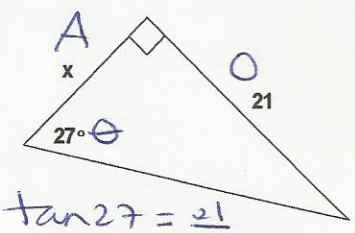


$\sin 32 = \frac{x}{15}$

$x = 15\sin 32$

$x = 7.9$

b.)

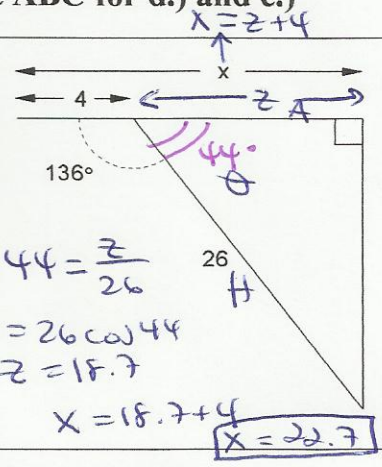


$\tan 27 = \frac{21}{x}$

$x = \frac{21}{\tan 27}$

$x = 41.2$

c.)



$\cos 44 = \frac{z}{26}$

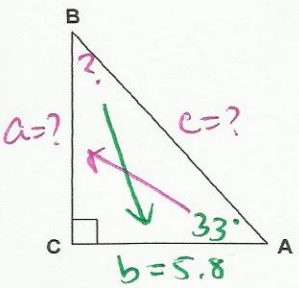
$z = 26\cos 44$

$z = 18.7$

$x = 18.7 + 4$

$x = 22.7$

d.) Solve triangle ABC: A = 33°, b = 5.8 (3 answers)



$B = 90 - 33$

$B = 57°$

$\tan 33 = \frac{a}{5.8}$

$a = 5.8\tan 33$

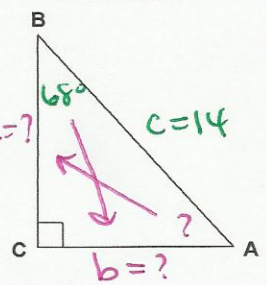
$a = 3.8$

$3.8^2 + 5.8^2 = c^2$

$\sqrt{48.08} = \sqrt{c^2}$

$c = 6.9$

e.) Solve triangle ABC: B = 68°, c = 14 (3 answers)



$A = 90 - 68$

$A = 22°$

$\cos 68 = \frac{a}{14}$

$a = 14\cos 68$

$a = 5.2$

$5.2^2 + b^2 = 14^2$

$-5.2^2 \quad -5.2^2$

$\sqrt{b^2} = \sqrt{168.96}$

$b = 13$