Angles, UC, Trig Graphs/Equations – Solving Trigonometric Equations

Steps For Solving Trigonometric Equations

> Example: $\sin x(\cos x - 1) = 0$
>
> Do NOT do → $\dfrac{\sin x(\cos x - 1)}{\sin x} = \dfrac{0}{\sin x}$
>
> Do do → $\sin x = 0$ and $\cos x - 1 = 0$

1.) Simplify the equation like any other equation such as…

- Get rid of parentheses by using the distributive property
- Collect like terms and isolate the trigonometric function (which contains x) on one side
- If the trigonometric function is already in factored form → SET EACH FACTOR = 0 and solve them! Do NOT divide out each side by a trig expression!
- Cross multiply if have (or get) the equation to have a ratio (fraction) on either side of equal sign.

2.) Many equations will have MULTIPLE SOLUTIONS (ANGLES) where some (or all) don't check → so you can have… a.) 1, 2, 3, or 4 solutions b.) some extraneous solution(s) c.) no solution

3.) Make sure you know what QUADRANTS sine, cosine, and tangent are POSITIVE and NEGATIVE. Refer to your Trig Chart/Unit Circle Sheet and where to obtain some multiple answers (angles).

**Notes:

- Your solution(s) will need to be between 0° and 360° so some angles may not "fit", any angle that doesn't "fit" this rule is considered an extraneous solution (doesn't fit/work/check).
- You are ALLOWED to add 360° to make a negative angle become positive BUT you are NOT ALLOWED to subtract 360° to make positive angle "fit" the rule.
- You must put final solution(s) in ascending order, do not include any extraneous solutions, and do not rewrite solutions that are repeated.

Example 1: Solve each trigonometric equation. Keep your answer(s) in degrees where $0° \le x < 360°$.

a.) $\cos x + \cos x = \sqrt{2}$

$\dfrac{2\cos x}{2} = \dfrac{\sqrt{2}}{2}$

$\cos^{-1}\left(\cos x = \dfrac{\sqrt{2}}{2}\right)\cos^{-1}$

$x = \cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

$x = 45°, 315°$

Solution(s): 45°, 315°

b.) $3\sin x - 2 = 5\sin x - 1$

$\cancel{-3\sin x} + 1 \quad -3\sin x + 1$

$\dfrac{-1}{2} = \dfrac{2\sin x}{2}$

$\sin^{-1}\left(\sin x = -\dfrac{1}{2}\right)\sin^{-1}$

$x = \sin^{-1}\left(-\dfrac{1}{2}\right)$

$x = -30 + 360$

$x = 330°, 210°$

Solution(s): 210°, 330°

c.) $\tan x\left(2\sin x - \sqrt{3}\right) = 0$

$\tan^{-1}(\tan x = 0)\tan^{-1} \qquad 2\sin x - \sqrt{3} = 0$

$x = \tan^{-1}(0) \qquad \dfrac{2\sin x}{2} = \dfrac{\sqrt{3}}{2}$

$x = 0°, 180° \qquad x = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$x = 60°, 120°$

Solution(s): 0°, 60°, 120°, 180°

d.) $2\tan 2x - 8\tan 2x = 6\sqrt{3}$

$\dfrac{-6\tan 2x}{-6} = \dfrac{6\sqrt{3}}{-6}$

$\tan^{-1}\left(\tan 2x = -\sqrt{3}\right)\tan^{-1}$

$2x = \tan^{-1}\left(-\sqrt{3}\right)$

$2x = -60 + 360$

$\dfrac{2x}{2} = \dfrac{300°}{2}, \dfrac{120}{2}$

$x = 150°, 60°$

Solution(s): 60°, 150°

e.) $3\cos x = 6 - 2(1 - \cos x)$

$3\cos x = 6 - 2 + 2\cos x$

$3\cos x = 4 + 2\cos x$

$-2\cos x \qquad -2\cos x$

$\cos^{-1}(\cos x = 4)\cos^{-1}$

$\emptyset$ (ex sol)

Solution(s): no solution

f.) $\dfrac{1}{\sin x} = \dfrac{2}{\sqrt{2}}$

$\dfrac{2\sin x}{2} = \dfrac{\sqrt{2}}{2}$

$\sin x = \dfrac{\sqrt{2}}{2}$

$x = \sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

$x = 45°, 225°$

Solution(s): 45°, 225°

g.) $(2\cos x - 1)(\sin x + 1) = 0$

$2\cos x - 1 = 0 \qquad \sin x + 1 = 0$

$2\cos x = 1 \qquad \sin x = -1$

$\cos x = \dfrac{1}{2} \qquad x = \sin^{-1}(-1)$

$x = \cos^{-1}\left(\dfrac{1}{2}\right) \qquad x = -90° + 360$

$x = 60°, 300° \qquad x = 270°$

Solution(s): 60, 270, 300

h.) $8\cos\left(\dfrac{3}{7}x\right) + 4\sqrt{3} = 0$

$\dfrac{8\cos\left(\frac{3}{7}x\right)}{8} = \dfrac{-4\sqrt{3}}{8}$

$\cos\left(\dfrac{3}{7}x\right) = \dfrac{\sqrt{3}}{-2}$

$\dfrac{3}{7}x = \cos^{-1}\left(\dfrac{\sqrt{3}}{-2}\right)$

$7\left(\dfrac{3}{7}x = 150°, 210°\right)7$

$3x = 1050, 1470$

$x = 350°, 490° \to \emptyset$

Solution(s): 350°

$y = a\sin(b^\circ t + c) + d$
$y = a\cos(b^\circ t + c) + d$

Example 2: Complete each application problem involving trigonometric equations.

a.) The tide cycle of a city on the Atlantic coast can be represented by the equation of $h = 9\sin(30^\circ t)$ where h = height of the tide in feet and t = number of hours since the last high tide. A tide is at equilibrium when it's at its normal level, halfway between its highest and lowest points. How many hours will it take for the first low tide to reach 3 feet? $h = -3$, $t = ?$

$$\frac{-3}{9} = \frac{9\sin(30^\circ t)}{9}$$

$$-\frac{1}{3} = \sin(30^\circ t)$$

$$\sin^{-1}\left(-\tfrac{1}{3}\right) = 30^\circ t$$

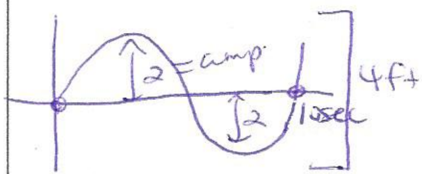
$$-19.5^\circ = 30^\circ t$$

+360

$$\frac{340.5^\circ}{30} = \frac{30^\circ t}{30}$$

$$\boxed{t = 11.4\text{ hrs}}$$

b.) A buoy in the harbor of San Juan, Puerto Rico, bobs up and down. The distance between the highest and lowest point is 4 feet. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. What is the height of the buoy after 3 seconds? $t = 3$, $h = ?$



amp = 2

period = 10 seconds.

$a = 2$

$b = \frac{2\pi}{10} = \frac{\pi}{5} = 36^\circ$

$$h = a\sin(b^\circ t)$$

$$h = 2\sin(36^\circ t)$$

$$h = 2\sin(36^\circ \cdot 3)$$

$$\boxed{h = 1.9\text{ ft}}$$

c.) As you ride a Ferris wheel, the height that you are above the ground varies periodically as a function of time. Jason gets into a seat that is at the bottom of the Ferris wheel, he is 3 feet above the ground. The wheel has a diameter of 32 feet and it takes the wheel 18 seconds to complete one cycle. How high above the ground will Jason be after 8 seconds riding the wheel?

part of vertical shift

amplitude

period

$t = 8$ $h = ?$

$$h = a\sin(b^\circ t) + d$$

amp $= \frac{32}{2} = 16 \rightarrow a = 16$

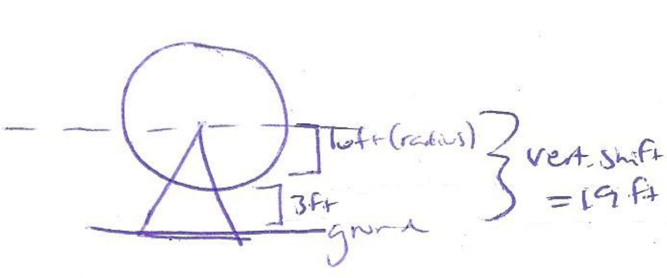
period $= \frac{2\pi}{18} = \frac{\pi}{9} \rightarrow b = 20^\circ$

vert. shift = 3ft + radius (16) $\rightarrow d = 19$

$$h = 16\sin(20^\circ t) + 19$$

$$h = 16\sin(20^\circ \cdot 8) + 19$$

$$\boxed{h = 24.5\text{ ft}}$$



d.) In a certain wildlife refuge, the population of field mice can be modeled by the equation $h = 3000 - 1250\cos(120^\circ t + 4)$ where h = the number of mice and t = the number of months past March 1 of a given year. About what date will the number of mice reach its maximum amount?

d vert shift

amp a

maximum amt = |ampl| + vert. shift

$= |-1250| + 3000 = 4250$

$$4250 = 3000 - 1250\cos(120t + 4)$$
$$-3000 \qquad -3000$$

$$\frac{1250}{-1250} = \frac{-1250\cos(120t+4)}{-1250}$$

$$-1 = \cos(120t + 4)$$

$$\cos^{-1}(-1) = 120t + 4$$

$$180 = 120^\circ t + 4$$

$$\frac{176}{120} = \frac{120^\circ t}{120}$$

$t = 1.5$ months.

March 1 + 1.5 months

$= \boxed{\text{April 15th}}$