Advanced Functions – Review for Unit # 1 Test WS Name: Key

Complete each problem about each arithmetic sequence/series or geometric sequence/series. Must show your work to receive credit!!

1.) Find the 20^{th} term for the arithmetic sequence in which $a_1 = 3$ and $d = 7$.

$a_{20} = 3 + 7(20-1)$

$a_{20} = 3 + 7(19)$

$\boxed{a_{20} = 136}$

2.) If the 15^{th} term for an arithmetic sequence is – 76 and the common difference is – 6, then find the fifth term.

① $-76 = a_1 - 6(15-1)$

$-76 = a_1 - 84$

$+84 \quad +84$

$a_1 = 8$

② $a_5 = 8 - 6(5-1)$

$\boxed{a_5 = -16}$

3.) Which term of the arithmetic sequence 5 , 8 , 11 , …. is 128?

$128 = 5 + 3(n-1)$

$-5 \quad -5$

$123 = 3n - 3$

$+3 \quad +3$

$126 = 3n$

$\frac{126}{3} = \frac{3n}{3}$

$n = 42 \rightarrow$ $\boxed{42^{nd} \text{ term}}$

4.) The eighteenth term is 56 and the twenty-seventh term is 92 for an arithmetic sequence, what is the eight term?

① $56 = a_1 + 17d$

$-92 = a_1 + 26d$

$-36 = -9d$

$d = 4$

② $56 = a_1 + 17(4)$

$-68 \quad -68$

$a_1 = -12$

③ $a_8 = -12 + 4(8-1)$

$\boxed{a_8 = 16}$

5.) Find the two arithmetic means between 4 and 22.

4, 10, 16, 22 ✓

$a_1 \quad a_2 \quad a_3 \quad a_4$

$22 = 4 + d(4-1)$

$22 = 4 + 3d$

$-4 \quad -4$

$18 = 3d$

$d = 6$

6.) Find the sum of the first 26 terms for an arithmetic series in which $a_1 = 3$ and the last term is 15.

$S_{26} = \frac{26}{2}(3 + 15)$

$S_{26} = 13(18)$

$\boxed{S_{26} = 234}$

7.) If $S_{19} = -760$ and $a_1 = 14$, then find the common difference.

① $-760 = \frac{19}{2}(14 + a_{19}) \cdot 2$

$-1520 = 266 + 19a_{19}$

$-1786 = 19a_{19}$

$a_{19} = -94$

② $-94 = 14 + d(19-1)$

$-94 = 14 + 18d$

$-108 = 18d$

$\boxed{d = -6}$

8.) If the sum of an arithmetic series with 24 terms is 1872 and the last term in the series is 147, then find the first term of the series.

$1872 = \frac{24}{2}(a_1 + 147)$

$1872 = 12(a_1 + 147)$

$1872 = 12a_1 + 1764$

$-1764 \quad -1764$

$108 = 12a_1$

$\boxed{a_1 = 9}$

9.) Find sum using any method:

a.) $\sum_{n=3}^{6} 4n + 5$ $a_1 = 4(3) + 5 = 17$, $a_n = 4(6) + 5 = 29$, $n = 6 - 3 + 1 = 4$

$S_4 = \frac{4}{2}(17 + 29)$

$\boxed{S_4 = 92}$

b.) $\sum_{n=1}^{7} 5(-4)^{n-1}$ $a_1 = 5(-4)^{1-1} = 5$, $r = \frac{-4}{5} = -4$, $n = 7 - 1 + 1 = 7$

$S_7 = \frac{5(1-(-4)^7)}{(1-(-4))}$

$\boxed{S_7 = 6385}$

c.) $\sum_{n=1}^{\infty} 20\left(-\frac{3}{2}\right)^{n-1}$ $a_1 = 20$, $r = -\frac{3}{2}$

$\boxed{S = \text{DNE}}$

10.) Find the 6^{th} term of the geometric sequence for which $a_1 = 4$ and $r = -2$.

$a_6 = 4(-2)^{6-1}$

$a_6 = 4(-2)^5$

$\boxed{a_6 = -128}$

11.) If the seventh term of a geometric sequence is 3,645 and the common ratio is 3, then find the first term.

$3645 = a_1(3)^6$

$\frac{3645}{729} = \frac{a_1(729)}{729}$

$\boxed{a_1 = 5}$

12.) If the third term in a geometric sequence is $\frac{23}{2}$ and the sixth term is $-\frac{23}{16}$, then find the first term.

① $-\frac{23}{16} = a_1 r^5$

÷ ② $\frac{23}{2} = a_1 r^2$

$\sqrt[3]{-\frac{1}{8}} = \sqrt[3]{r^3}$

$r = -\frac{1}{2}$

② $\frac{23}{2} = a_1\left(-\frac{1}{2}\right)^2$

$4\left(\frac{23}{2}\right) = a_1\left(\frac{1}{4}\right) \cdot 4$

$\boxed{a_1 = 46}$

13.) Find the four geometric means between 5 and 1,215.

5, 15, 45, 135, 405, 1215
a_1 a_2 a_3 a_4 a_5 a_6

$\frac{1215}{5} = \frac{5(r)^5}{5}$

$\sqrt[5]{243} = \sqrt[5]{r^5}$

$r = 3$

14.) Find the sum of the (finite) geometric series $128 - 64 + 32 - \ldots$ to 8 terms.

a_1 ✓ $r = \frac{-64}{128} = -\frac{1}{2}$ ✓

$S_8 = \frac{128\left(1-\left(-\frac{1}{2}\right)^8\right)}{\left(1-\left(-\frac{1}{2}\right)\right)}$

$S_8 = 85$

15.) If $S_n = 5{,}115$, $r = 2$, and $a_1 = 5$, then find the number of terms of the (finite) geometric series.

$5115 = \frac{5(1-2^n)}{(1-2)}$

$5115 = -5(1-2^n)$

$-1023 = 1 - 2^n$

$-1024 = -2^n$

$1024 = 2^n$ → $\frac{\log 1024}{\log 2} = \frac{n \log 2}{\log 2}$

$n = 10$ → 10 terms

16.) Find the common ratio for an infinite geometric series in which $a_1 = 6$ and $S = 18$.

$18 = \frac{6}{1-r}$

$18 - 18r = 6$

$\frac{-18r}{-18} = \frac{-12}{-18}$

$r = \frac{2}{3}$

17.) Find the sum (if it exists) for an infinite geometric series

$4 + 3 + \frac{9}{4} + \ldots$ (a_1)

$r = \frac{3}{4} = .75$ ✓

$S = \frac{4}{\left(1-\left(\frac{3}{4}\right)\right)}$

$S = 16$

18.) Write the following repeating decimal as a fraction: 0.648484848....

$.6 + [.048 + .00048 + \ldots]$

↙ not a series; infinite series

$a_1 = .048$

$r = \frac{.00048}{.048} = .01$

$S = \frac{.048}{(1-.01)} + .6 = \frac{107}{165}$

19.) Find the fifth term of the sequence in which $a_1 = 4$ and $a_{n+1} = 4a_n + 6$.

$a_1 = 4$

$a_2 = 4(4) + 6 = 22$

$a_3 = 4(22) + 6 = 94$

$a_4 = 4(94) + 6 = 382$

$a_5 = 4(382) + 6 = 1534$

20.) Find the third iterate of $f(x) = 3x - 5$ for an initial value of $x_0 = 2$.

$x_0 = 2$

$x_1 = 3(2) - 5 = 1$

$x_2 = 3(1) - 5 = -2$

$x_3 = 3(-2) - 5 = -11$

21.) What is the twelfth term of the Fibonacci sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

$a_{12} = 144$

22.) The grocery store manager, Mr. Whipple, asked George to make a pyramid display of cans of chicken soup that came in the last shipment. George stacked 76 cans on the bottom. By removing two cans from each end, there are only eight cans left for the top row. How many cans are in the entire display?

$a_1 = 76$, $d = -4$, $a_n = 8$, $S_n = ?$

① $8 = 76 - 4(n-1)$

$-68 = -4n + 4$

$-72 = -4n$

$n = 18$

② $S_{18} = \frac{18}{2}(76+8)$

$= 756$ cans

23.) A researcher placed four single celled organisms in a laboratory petri dish. Overnight, each cell split into two; the next night each of those cells split into two, etc. Assuming no deaths, after how many weeks will the petri dish have 4,194,304 organisms?

$a_1 = 4$, $r = 2$, $a_n = 4194304$, $n = ?$

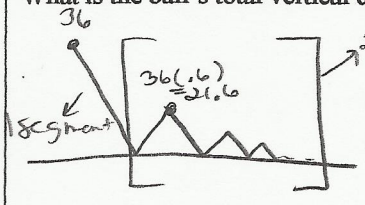
$\frac{4194304}{4} = \frac{4(2)^{n-1}}{4}$

$1048576 = (2)^{n-1}$

$1 + \frac{\log 1048576}{\log 2} = (n-1)\frac{\log 2}{\log 2} + 1$

$n = 21$ days → 3 weeks

24.) A rubber ball is dropped from 36 ft. It rebounds to a height of 60% of the drop where this process continues infinitely. What is the ball's total vertical distance?



$S = \left(\frac{21.6}{1-.6}\right)2 + 36$

$= 144$ ft

25.) The Dawn family is taking out a mortgage loan for $180,000 to buy a house. Their monthly payment is $878.79. The recursive formula is $a_n = (1.004375 \cdot a_{n-1}) - 878.79$ which describes the balance left on the loan after n payments. Find the balance of the loan after the fourth payment.

$a_1 = (1.004375 \cdot 180000) - 878.79 = 179908.71$

$a_2 = (1.004375 \cdot 179908.71) - 878.79 = 179817.02$

$a_3 = (1.004375 \cdot 179817.02) - 878.79 = 179724.93$

$a_4 = (1.004375 \cdot 179724.93) - 878.79 = \$179{,}632.44$