Advanced Functions – Review for Geometric Quiz WS Name: Key

Directions – Each problem below is geometric (finite or infinite). Complete and show your work!

1.) What is the explicit (n^{th} term) formula for the geometric sequence of 16 , 80 , 400 , …?

$a_n = a_1(r)^{n-1}$ $a_1 = 16$ $r = \frac{80}{16} = 5$

→ $\boxed{a_n = 16(5)^{n-1}}$

2.) What is the fifth term of a geometric sequence whose first term is 32,768 and common ratio is one fourth?

$a_1 = 32768$ $r = \frac{1}{4} \rightarrow a_5 = ?$

$a_5 = 32768\left(\frac{1}{4}\right)^4$

$\boxed{a_5 = 128}$

3.) The seventh term of a geometric sequence is – 234,375 and the common ratio is – 5. What is the first term?

$a_7 = -234375$ $r = -5 \rightarrow a_1 = ?$

$-234375 = a_1(-5)^6$

$\frac{-234375}{15625} = \frac{a_1(15625)}{15625}$

$\boxed{a_1 = -15}$

4.) What are four geometric means for sequence 3 , 12 , 48 , 192 , 768 , 3072?

a_1 a_2 a_3 a_4 a_5 a_6

$\frac{3072}{3} = \frac{3(r)^5}{3}$

$\sqrt[5]{1024} = \sqrt[5]{r^5}$

$r = 4$

5.) What is the sum of first four terms of a finite geometric series whose first term is 156 and the common ratio is – 2?

$a_1 = 156$ $r = -2$ $n = 4 \rightarrow S_4 = ?$

$S_4 = \frac{156(1-(-2)^4)}{(1-(-2))}$

$\boxed{S_4 = -780}$

6.) What is the sum of the finite geometric series $2 + 12 + 72 + \ldots + 93{,}312$?

a_1 $r = \frac{12}{2} = 6$ a_n ① $n = ?$ ② $S_n = ?$

① $\frac{93312}{2} = \frac{2(6)^{n-1}}{2}$

$46656 = 6^{n-1}$

$\frac{\log 46656}{\log 6} = \frac{(n-1)\log 6}{\log 6} + 1$

$n = 7$

② $S_7 = \frac{2(1-6^7)}{(1-6)}$

$\boxed{S_7 = 111974}$

7.) The sum of first 8 terms of a finite geometric series is – 16,400 whose common ratio is 3. What is the first term of the series?

$S_8 = -16400$ $n = 8$ $r = 3 \rightarrow a_1 = ?$

$-2(-16400) = \frac{a_1(1-3^8)}{(1-3)} \cdot -2$

$\frac{32800}{-6560} = \frac{a_1(-6560)}{-6560}$

$\boxed{a_1 = -5}$

8.) Determine if each infinite geometric series is convergent (if so, then find sum) or divergent:

↓ $-1 < r < 1$

a.) $144 + 36 + 9 \ldots$ $r = \frac{36}{144} = .25$ ✓ $S = \frac{144}{(1-.25)}$

Converges to 192 $S = 192$

b.) $-2 - 10 - 50 - \ldots$ $r = \frac{-10}{-2} = 5$ ✗ $S = DNE$

diverges

c.) $4 + 6 + 9 + \ldots$ $r = \frac{6}{4} = 1.5$ ✗ $S = DNE$

diverges

d.) $6 - 2 + \frac{2}{3} - \ldots$ $r = \frac{-2}{6} = -\frac{1}{3}$ ✓ $S = \frac{6}{(1-(-\frac{1}{3}))}$

Converges to $\frac{9}{2}$ $S = \frac{9}{2}$

9.) What is the common ratio of an infinite geometric series whose first term is 14 and sum is 56?

$S = 56$ $a_1 = 14 \rightarrow r = ?$

$\frac{56}{1} = \frac{14}{1-r}$

$56 - 56r = 14$

$\frac{-56r}{-56} = \frac{-42}{-56}$

$\boxed{r = \frac{3}{4}}$

10.) Find the sum (if it exists):

$a_1 = 3(-2)^{3-1} = 12$ $S_4 = -60$ ✓

a.) $\sum_{n=3}^{6} 3(-2)^{n-1}$ $n = 6-3+1$, $n = 4$

$S_4 = \frac{12(1-(-2)^4)}{(1-(-2))}$

b.) $\sum_{n=1}^{\infty} -4\left(\frac{3}{5}\right)^{n-1}$ $S = \frac{-4}{(1-(\frac{3}{5}))}$

$\boxed{S = -10}$

c.) $\sum_{n=5}^{12} 2\left(\frac{7}{2}\right)^{n-1}$ $n = 12-5+1$, $n = 8$

$S_8 = \frac{300.125(1-(\frac{7}{2}))}{(1-(\frac{7}{2}))}$

$\boxed{S_8 = 2703256.375}$

$a_1 = 2(\frac{7}{2})^{5-1} = 300.125$

d.) $\sum_{n=1}^{\infty} 5\left(\frac{7}{4}\right)^{n-1}$ 1.75 ✗

$\boxed{S = DNE}$