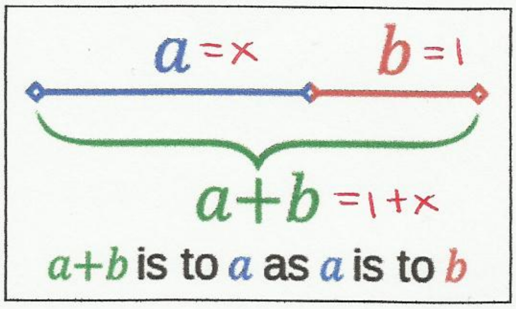
Sequences and Series – Recursive and Special Sequences

Specific Sequence # 3 – Fibonacci Sequence

The following terms are in the Fibonacci Sequence → 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , __?__ …

- What is the pattern to this sequence? the sum of the two previous terms
- What is the missing term (value of the ?) to this sequence? $55+89=144$ → $\boxed{?=144}$
- The Golden Ratio is a phenomena that is associated with the Fibonacci Sequence →



The Golden Ratio actually equals a specific number, let's figure it out:

$$\frac{1+x}{x}=\frac{x}{1} \rightarrow 1+x=x^2$$

(cross multiply)

$$-1\ -x \qquad -1-x$$

$$x^2-x-1=0$$

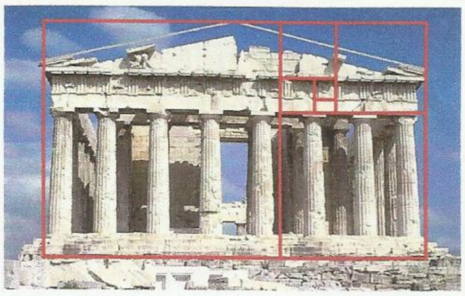
$a=1$, $b=-1$, $c=-1$ } Quad Formula

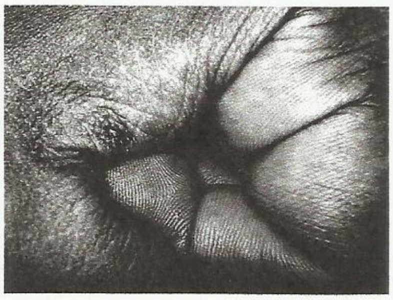
$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

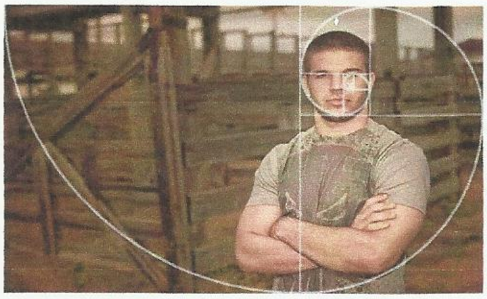
$$\frac{1\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$$

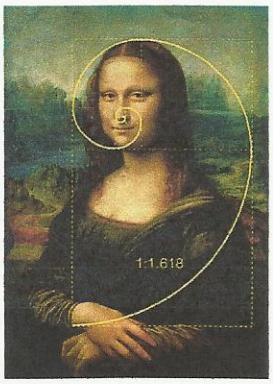
$$\frac{1\pm\sqrt{5}}{2}\approx 1.618033989 \rightarrow \varphi \text{ (phi)}$$

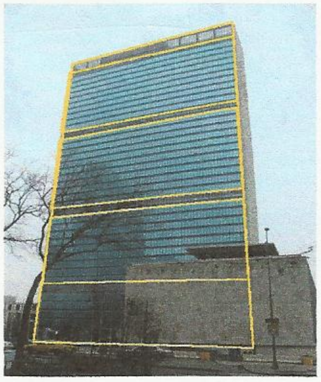
- Both the Fibonacci Sequence and the Golden Ratio occur naturally and humanly in the real world:

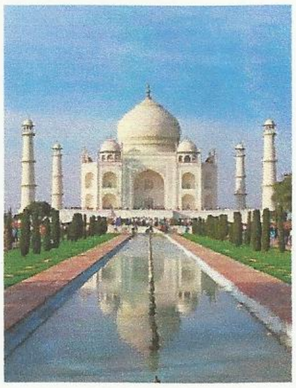












Specific Sequence # 4 – Recursive Sequence (Formula)

- **recursive sequence** → a sequence that uses a previos term to get to the next term. where the sequence is neither consistently adding/subtracting (like arithmetic sequences) nor is the sequence consistently multiplying/dividing (like geometric sequences)

Note: The following are examples of recursive sequences → $a_n = 2a_{n-1} + 5$ or $a_{n+1} = 2a_n + 5$

Example 1: Find the first five terms of the given sequence.

a.) $a_{n+1} = 3a_n - 2$; $a_1 = 4$	b.) $a_n = -2(a_{n-1} + 6)$; $a_1 = 3$	c.) $a_{n+1} = 4a_n + 2n$; $a_1 = 5$
$a_1 = 4$ $(n=1)\ a_{1+1} = 3a_1 - 2 \rightarrow a_2 = 3(4) - 2$ $a_2 = 10$ $(n=2)\ a_{2+1} = 3a_2 - 2 \rightarrow a_3 = 3(10) - 2$ $a_3 = 28$ $(n=3)\ a_{3+1} = 3a_3 - 2 \rightarrow a_4 = 3(28) - 2$ $a_4 = 82$ $(n=4)\ a_{4+1} = 3a_4 - 2 \rightarrow a_5 = 3(82) - 2$ $a_5 = 244$ $\boxed{4, 10, 28, 82, 244, \ldots}$	$a_1 = 3$ $(n=2)\ a_2 = -2(a_{2-1} + 6) = -2(a_1 + 6)$ $= -2(3+6) = -18 = a_2$ $(n=3)\ a_3 = -2(a_{3-1} + 6) = -2(a_2 + 6)$ $= -2(-18+6) = 24 = a_3$ $(n=4)\ a_4 = -2(a_{4-1} + 6) = -2(a_3 + 6)$ $= -2(24+6) = -60 = a_4$ $(n=5)\ a_5 = -2(a_{5-1} + 6) = -2(a_4 + 6)$ $= -2(-60+6) = 108 = a_5$ $\boxed{3, -18, 24, -60, 108, \ldots}$	$a_1 = 5$ $(n=1)\ a_{1+1} = 4a_1 + 2(1)$ $= 4(5) + 2 = 22 = a_2$ $(n=2)\ a_{2+1} = 4a_2 + 2(2)$ $= 4(22) + 4 = 92 = a_3$ $(n=3)\ a_{3+1} = 4a_3 + 2(3)$ $= 4(92) + 6 = 374 = a_4$ $(n=4)\ a_{4+1} = 4a_4 + 2(4)$ $= 4(374) + 8 = 1504 = a_5$ $\boxed{5, 22, 92, 374, 1504, \ldots}$

Example 2: Complete the word problem below using recursive sequences.

Caleb discovered that there were 225 dandelions in his garden on the first Saturday of spring. He had time to pull out 100, but by the next Saturday, there were twice as many as he had left. Each Saturday in spring, he removed 100 dandelions, only to find that the number of remaining dandelions had doubled by the following Saturday.

a.) Write a recursive formula for the number of dandelions Caleb finds in his garden each Saturday.	b.) Find the number of dandelions Caleb would find on the ~~fourth~~ Saturday. $d_1 = 225$
d_n = # of dandelions at begin. of nth Sat. Caleb will pull out 100 → $d_n - 100$ d_{n+1} = # of dandelions of next Sat but will be twice as much $d_{n+1} = 2(d_n - 100)$ $\boxed{d_{n+1} = 2d_n - 200}$	$(n=1)\ d_{1+1} = 2d_1 - 200 = 2(225) - 200 = 250 = d_2$ $(n=2)\ d_{2+1} = 2d_2 - 200 = 2(250) - 200 = 300 = d_3$ $(n=3)\ d_{3+1} = 2d_3 - 200 = 2(300) - 200 = 400 = d_4$ There will be 400 dandelions on 4th Saturday

- **iteration** → the process of Composing a function with itself repeatedly.

Example 3: Find the first three terms x_1, x_2, and x_3 of the function $f(x) = 2x + 3$ where $x_0 = 1$.

$x_1 = f(x_0)$	$x_2 = f(x_1)$	$x_3 = f(x_2)$
$x_1 = f(1)$	$x_2 = f(5)$	$x_3 = f(13)$
$x_1 = 2(1) + 3 = 5$	$x_2 = 2(5) + 3 = 13$	$x_3 = 2(13) + 3 = 29$

$\boxed{1, 5, 13, 29, \ldots}$