

## Adv. Functions - Recursive Sequences WS.

1)  $a_{n+1} = 6 - a_n, a_1 = -4$

$(n=1) \rightarrow a_{1+1} = 6 - a_1$

$a_2 = 6 - (-4)$

$a_2 = 10$

$(n=3) \rightarrow a_{3+1} = 6 - a_3$

$a_4 = 6 - (-4)$

$a_4 = 10$

$(n=2) \rightarrow a_{2+1} = 6 - a_2$

$a_3 = 6 - 10$

$a_3 = -4$

$(n=4) \rightarrow a_{4+1} = 6 - a_4$

$a_5 = 6 - 10$

$a_5 = -4$

$-4, 10, -4, 10, -4, \dots$

2)  $a_{n+1} = 2a_n + 7, a_1 = -3$

$(n=1) \rightarrow a_{1+1} = 2a_1 + 7$

$a_2 = 2(-3) + 7$

$a_2 = 1$

$(n=3) \rightarrow a_{3+1} = 2a_3 + 7$

$a_4 = 2(9) + 7$

$a_4 = 25$

$(n=2) \rightarrow a_{2+1} = 2a_2 + 7$

$a_3 = 2(1) + 7$

$a_3 = 9$

$(n=4) \rightarrow a_{4+1} = 2a_4 + 7$

$a_5 = 2(25) + 7$

$a_5 = 57$

$-3, 1, 9, 25, 57, \dots$

3)  $a_{n+1} = a_n - 3n + 1, a_1 = 5$

$(n=1) \rightarrow a_{1+1} = a_1 - 3(1) + 1$

$a_2 = 5 - 3 + 1$

$a_2 = 3$

$(n=3) \rightarrow a_{3+1} = a_3 - 3(3) + 1$

$a_4 = 2 - 9 + 1$

$a_4 = -10$

$(n=2) \rightarrow a_{2+1} = a_2 - 3(2) + 1$

$a_3 = 3 - 6 + 1$

$a_3 = -2$

$(n=4) \rightarrow a_{4+1} = a_4 - 3(4) + 1$

$a_5 = -10 - 12 + 1$

$a_5 = -21$

$5, 3, -2, -10, -21, \dots$

$$4) a_n = 4a_{n-1} - 2, a_1 = 2$$

$$(n=2) \rightarrow a_2 = 4a_{2-1} - 2$$

$$a_2 = 4a_1 - 2$$

$$a_2 = 4(2) - 2$$

$$a_2 = 6$$

$$(n=3) \rightarrow a_3 = 4a_{3-1} - 2$$

$$a_3 = 4a_2 - 2$$

$$a_3 = 4(6) - 2$$

$$a_3 = 22$$

$$(n=4) \rightarrow a_4 = 4a_{4-1} - 2$$

$$a_4 = 4a_3 - 2$$

$$a_4 = 4(22) - 2$$

$$a_4 = 80$$

$$(n=5) \rightarrow a_5 = 4a_{5-1} - 2$$

$$a_5 = 4a_4 - 2$$

$$a_5 = 4(80) - 2$$

$$a_5 = 318$$

$$2, 6, 22, 80, 318, \dots$$

$$5) a_n = 3(a_{n-1} - 2), a_1 = 7$$

$$(n=2) \rightarrow a_2 = 3(a_{2-1} - 2)$$

$$a_2 = 3(a_1 - 2)$$

$$a_2 = 3(7 - 2)$$

$$a_2 = 15$$

$$(n=4) \rightarrow a_4 = 3(a_{4-1} - 2)$$

$$a_4 = 3(a_3 - 2)$$

$$a_4 = 3(39 - 2)$$

$$a_4 = 111$$

$$(n=3) \rightarrow a_3 = 3(a_{3-1} - 2)$$

$$a_3 = 3(a_2 - 2)$$

$$a_3 = 3(15 - 2)$$

$$a_3 = 39$$

$$(n=5) \rightarrow a_5 = 3(a_{5-1} - 2)$$

$$a_5 = 3(a_4 - 2)$$

$$a_5 = 3(111 - 2)$$

$$a_5 = 327$$

$$7, 15, 39, 111, 327, \dots$$

$$6) a_n = 4 - (a_{n-1} + 3), a_1 = -2$$

$$(n=2) \rightarrow a_2 = 4 - (a_{2-1} + 3)$$

$$a_2 = 4 - (a_1 + 3)$$

$$a_2 = 4 - (-2 + 3)$$

$$a_2 = 3$$

$$(n=3) \rightarrow a_3 = 4 - (a_{3-1} + 3)$$

$$a_3 = 4 - (a_2 + 3)$$

$$a_3 = 4 - (3 + 3)$$

$$a_3 = -2$$

$$(n=4) \rightarrow a_4 = 4 - (a_{4-1} + 3)$$

$$a_4 = 4 - (a_3 + 3)$$

$$a_4 = 4 - (-2 + 3)$$

$$a_4 = 3$$

$$(n=5) \rightarrow a_5 = 4 - (a_{5-1} + 3)$$

$$a_5 = 4 - (a_4 + 3)$$

$$a_5 = 4 - (3 + 3)$$

$$a_5 = -2$$

$$-2, 3, -2, 3, -2, \dots$$



$$7) a_{n+1} = a_n - a_{n-1}, a_1 = -1, a_2 = -1$$

$$(n=2) \rightarrow a_{2+1} = a_2 - a_{2-1} \quad (n=3) \rightarrow a_{3+1} = a_3 - a_{3-1} \quad (n=4) \rightarrow a_{4+1} = a_4 - a_{4-1}$$

$$a_3 = a_2 - a_1$$

$$a_4 = a_3 - a_2$$

$$a_5 = a_4 - a_3$$

$$a_3 = -1 - (-1)$$

$$a_4 = 0 - (-1)$$

$$a_5 = 1 - 0$$

$$a_3 = 0$$

$$a_4 = 1$$

$$a_5 = 1$$

$$-1, -1, 0, 1, 1, \dots$$

$$8) a_{n+1} = -4a_n + a_{n-1}, a_1 = 3, a_2 = -5$$

$$(n=2) \rightarrow a_{2+1} = -4a_2 + a_{2-1} \quad (n=3) \rightarrow a_{3+1} = -4a_3 + a_{3-1} \quad (n=4) \rightarrow a_{4+1} = -4a_4 + a_{4-1}$$

$$a_3 = -4a_2 + a_1$$

$$a_4 = -4a_3 + a_2$$

$$a_5 = -4a_4 + a_3$$

$$a_3 = -4(-5) + 3$$

$$a_4 = -4(23) + (-5)$$

$$a_5 = -4(-97) + 23$$

$$a_3 = 23$$

$$a_4 = -97$$

$$a_5 = 411$$

$$3, -5, 23, -97, 411, \dots$$

$$9) a_n = 2a_{n-1} + 3a_{n-2}, a_1 = 1, a_2 = 2$$

$$(n=3) \rightarrow a_3 = 2a_{3-1} + 3a_{3-2}$$

$$(n=4) \rightarrow$$

$$(n=5) \rightarrow$$

$$a_3 = 2a_2 + 3a_1$$

$$a_4 = 2a_{4-1} + 3a_{4-2}$$

$$a_5 = 2a_{5-1} + 3a_{5-2}$$

$$a_3 = 2(2) + 3(1)$$

$$a_4 = 2a_3 + 3a_2$$

$$a_5 = 2a_4 + 3a_3$$

$$a_3 = 7$$

$$a_4 = 2(7) + 3(2)$$

$$a_5 = 2(20) + 3(7)$$

$$a_4 = 20$$

$$a_5 = 50$$

$$1, 2, 7, 20, 50, \dots$$

$$10) f(x) = 9x - 2, x_0 = 2$$

$$x_1 = f(x_0)$$

$$x_2 = f(x_1)$$

$$x_3 = f(x_2)$$

$$x_1 = f(2)$$

$$x_2 = f(16)$$

$$x_3 = f(142)$$

$$x_1 = 9(2) - 2$$

$$x_2 = 9(16) - 2$$

$$x_3 = 9(142) - 2$$

$$x_1 = 16$$

$$x_2 = 142$$

$$x_3 = 1276$$

$$11) f(x) = 5 - 2x^2, x_0 = -1$$

$$x_1 = f(x_0)$$

$$x_1 = f(-1)$$

$$x_1 = 5 - 2(-1)^2$$

$$x_1 = 3$$

$$x_2 = f(x_1)$$

$$x_2 = f(3)$$

$$x_2 = 5 - 2(3)^2$$

$$x_2 = -13$$

$$x_3 = f(x_2)$$

$$x_3 = f(-13)$$

$$x_3 = 5 - 2(-13)^2$$

$$x_3 = -333$$

$$12) f(x) = 3x^2 - 3x + 2, x_0 = \frac{1}{3}$$

$$x_1 = f(x_0)$$

$$x_1 = f\left(\frac{1}{3}\right)$$

$$x_1 = 3\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + 2$$

$$x_1 = \frac{4}{3}$$

$$x_2 = f(x_1)$$

$$x_2 = f\left(\frac{4}{3}\right)$$

$$x_2 = 3\left(\frac{4}{3}\right)^2 - 3\left(\frac{4}{3}\right) + 2$$

$$x_2 = \frac{10}{3}$$

$$x_3 = f(x_2)$$

$$x_3 = f\left(\frac{10}{3}\right)$$

$$x_3 = 3\left(\frac{10}{3}\right)^2 - 3\left(\frac{10}{3}\right) + 2$$

$$x_3 = \frac{76}{3}$$

$$13) a_n = (1.006 \cdot a_{n-1}) - 678.79, a_1 = 100,000$$

$$a_2 = (1.006 \cdot a_{2-1}) - 678.79$$

$$a_2 = (1.006 \cdot a_1) - 678.79$$

$$a_2 = (1.006 \cdot 100,000) - 678.79$$

$$a_2 = \$99,921.21 \rightarrow \text{1st payment}$$

$$a_3 = (1.006 \cdot a_{3-1}) - 678.79$$

$$a_3 = (1.006 \cdot a_2) - 678.79$$

$$a_3 = (1.006 \cdot 99,921.21) - 678.79$$

$$a_3 = \$99,841.95 \rightarrow \text{2nd payment}$$

$$a_4 = (1.006 \cdot a_{4-1}) - 678.79$$

$$a_4 = (1.006 \cdot a_3) - 678.79$$

$$a_4 = (1.006 \cdot 99,841.95) - 678.79$$

$$a_4 = \$99,762.21 \rightarrow \text{3rd payment}$$

$$a_5 = (1.006 \cdot a_{5-1}) - 678.79$$

$$a_5 = (1.006 \cdot a_4) - 678.79$$

$$a_5 = (1.006 \cdot 99,762.21) - 678.79$$

$$a_5 = \$99,681.99 \rightarrow \text{4th payment}$$



$$14) C(x) = 1.05x, X_0 = 2000$$

$$X_1 = f(x_0)$$

$$X_2 = f(x_1)$$

$$X_3 = f(x_2)$$

$$X_1 = f(2000)$$

$$X_2 = f(2100)$$

$$X_3 = f(2205)$$

$$X_1 = 1.05(2000)$$

$$X_2 = 1.05(2100)$$

$$X_3 = 1.05(2205)$$

$$X_1 = 2100$$

$$X_2 = 2205$$

$$X_3 = 2315.25$$

$$X_4 = f(x_3)$$

$$X_5 = f(x_4)$$

$$X_4 = f(2315.25)$$

$$X_5 = f(2431.01)$$

$$X_4 = 1.05(2315.25)$$

$$X_5 = 1.05(2431.01)$$

$$X_4 = 2431.01$$

$$X_5 = 2552.56$$