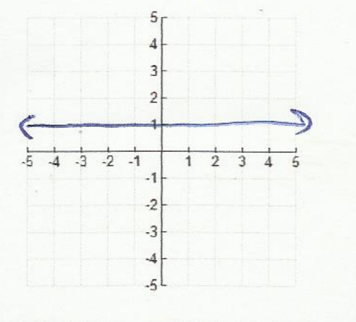
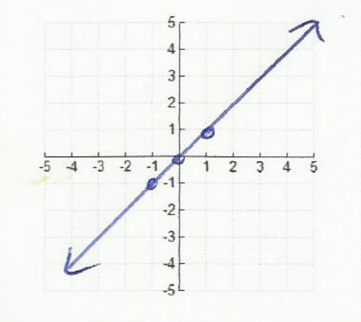
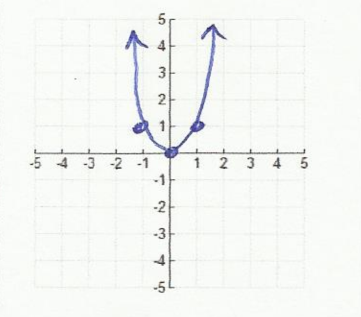
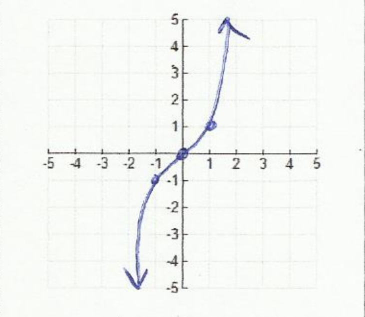
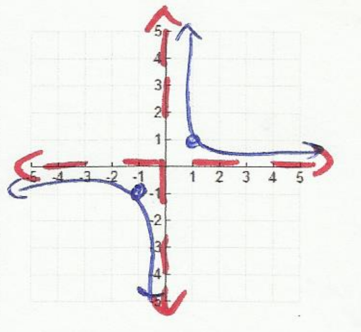
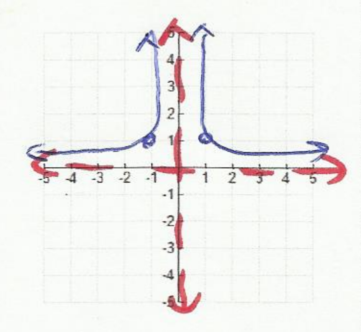
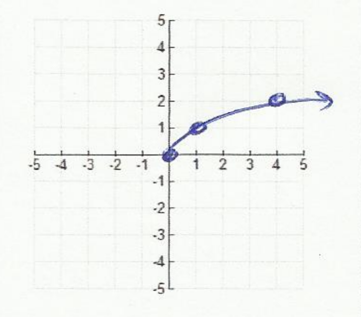
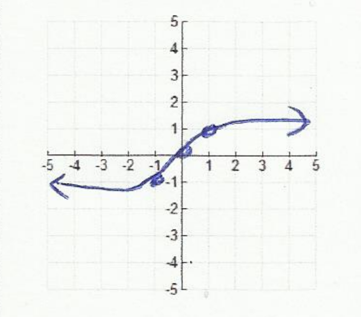
Functions/Regression – Power Functions and Equations

Power Functions and Their Characteristics

– **power function** → a function in the form $y = k \cdot x^p$ (k and p are constants) where k is called the constant of proportionality and p is the exponent (p can be any real value, rational/irrational, or positive/negative)

Below are some examples of power functions where k = 1 and various values for p:

Power Function # 1	Power Function # 2	Power Function # 3	Power Function # 4
$y = x^0 \rightarrow y = 1$	$y = x^1 \rightarrow y = x$	$y = x^2$ (or x^4, x^6, …)	$y = x^3$ (or x^5, x^7, …)
			
Power Function # 5	**Power Function # 6**	**Power Function # 7**	**Power Function # 8**
$y = x^{-1}$ or $y = 1/x$	$y = x^{-2}$ or $y = 1/x^2$	$y = x^{1/2}$ or $y = \sqrt{x}$	$y = x^{1/3}$ or $y = \sqrt[3]{x}$
			

Example 1: Determine which are power functions, circle YES or NO. If YES, state value of k and p.

a.) $f(x) = 5\sqrt[3]{x^{12}}$ power function? Circle one: YES NO where k = 5 and p = 4

$5x^{\frac{12}{3}} = 5x^4$

b.) $f(x) = 6(x-1)^2$ power function? Circle one: YES NO where k = X and p = X

c.) $f(x) = \sqrt{\dfrac{36}{x^5}} = \sqrt{36x^{-5}} = 6x^{-5/2}$ power function? Circle one: YES NO where k = 6 and p = $\frac{-5}{2}$

d.) $10y + 2 = 5x^4 + 2$ power function? Circle one: YES NO where k = $\frac{1}{2}$ and p = 4

$\frac{10y}{10} = \frac{5x^4}{10} = \frac{1}{2}x^4$

e.) $f(x) = -5 \cdot 2^x$ power function? Circle one: YES NO where k = X and p = X

f.) $\frac{1}{4}y = (x-3)(x+3) + 9$ power function? Circle one: YES NO where k = 4 and p = 2

$\frac{1}{4}y = x^2 - 9 + 9 \rightarrow \frac{1}{4}y = x^2 \rightarrow y = 4x^2$

g.) $y + 3 = 3(x+1)$ power function? Circle one: YES NO where k = 3 and p = 1

$y + 3 = 3x + 3$
$\;\; -3 \quad\;\; -3$

$y = 3x$

Example 2: Find the equation of a power function with the given information.

Power Function in the form $y = k \cdot x^p$ where the point (x , y) and the point (1 , ?) (k = ?) are on the graph		
a.) pts (2 , 12) ; (1 , 4) $\rightarrow k=4$ $y = 4x^p$ $\frac{12}{4} = \frac{4 \cdot 2^p}{4}$ $3 = 2^p$ $\log 3 = \log 2^p$ $\frac{\log 3}{\log 2} = \frac{p \log 2}{\log 2}$ $p = 1.585$ $\boxed{y = 4 \cdot x^{1.585}}$	b.) pts (7 , 9) ; (1 , 0.5) $\rightarrow k = .5$ $y = .5x^p$ $\frac{9}{.5} = \frac{.5(7)^p}{.5}$ $18 = 7^p$ $\log 18 = \log 7^p$ $\frac{\log 18}{\log 7} = \frac{p \cdot \log 7}{\log 7}$ $p = 1.485$ $\boxed{y = \frac{1}{2}x^{1.485}}$	c.) pts (4 , 0.375) and (9 , 0.25) $\frac{.375}{4^p} = \frac{k \cdot 4^p}{4^p}$ $k = \frac{.375}{4^p}$ $k = \frac{.375}{4^{.5}}$ $k = .75$ or $3/4$ $.25 = k \cdot 9^p$ $.25 = \left(\frac{.375}{4^p}\right) \cdot 9^p$ $\frac{.25}{.375} = \frac{.375\left(\frac{9}{4}\right)^p}{.375}$ $\frac{2}{3} = \left(\frac{9}{4}\right)^p$ $\frac{\log(2/3)}{\log(9/4)} = \frac{p \log(9/4)}{\log(9/4)}$ $p = -.5$ or $-1/2$ $y = \frac{3}{4}x^{-1/2} \rightarrow \boxed{y = \frac{3}{4\sqrt{x}}}$

Solving Power Equations – Direct and Inverse Variations

Direct Variation Equations: $y = k \cdot x^p$ (where p is a positive #)	**Inverse Variation Equations:** $y = k \cdot x^{-p}$ or $y = \frac{k}{x^p}$ (where p is a negative #)

Example 3: Complete each variation problem.

a.) Suppose y is directly proportional to x. If y = 18 when x = 8, find the constant of proportionality (k). After finding the formula for y, then use it to find x when y = 27. (understood exp of 1 since no mention of anything else) $y = k \cdot x$ $\frac{18}{8} = \frac{k \cdot 8}{8}$ $k = 2.25$ $\boxed{y = 2.25x}$ $\frac{27}{2.25} = \frac{2.25x}{2.25}$ $\boxed{x = 12}$	b.) Suppose c is inversely proportional to the square of d. If c = 4 when d = 2, find the constant of proportionality (k). After finding the formula for c, then use it to find c when d = – 8. $c = \frac{k}{d^2}$ $4 = \frac{k}{(2^2)}$ $4 = \frac{k}{4}$ $k = 16$ $\boxed{c = \frac{16}{d^2}}$ $c = \frac{16}{(-8)^2}$ $c = \frac{16}{64}$ $\boxed{c = \frac{1}{4}}$

c.) The radius of a sphere is directly proportional to the cube root of its volume. If a sphere of radius 18.2 cm has a volume of 25,252.4 cm^3, what is the volume of a sphere if the radius is 19.3 cm?

$r = k\sqrt[3]{V}$

$18.2 = k\sqrt[3]{25252.4}$

$\frac{18.2}{29.338} = \frac{29.338k}{29.338}$

$k = .62$

$\rightarrow r = .62\sqrt[3]{V}$

$\frac{19.3}{.62} = \frac{.62\sqrt[3]{V}}{.62}$

$(31.129)^3 = (\sqrt[3]{V})^3$

$\boxed{V = 30{,}164.45 \text{ cm}^3}$