

Functions/Regression – Piecewise Functions

- **piecewise function** → a function and its graph is broken into different parts (hence its name) and different functions are *defined on VARIOUS DOMAIN VALUES* (x-values).

Step # 1 → Evaluate the function based on its domain (x) values ; start with #'s after the word IF.

Step # 2 → Make a table of values (points) ; indicate if those points are closed or open dots.

Step # 3 → After graphing (by hand – calculator is not helpful), CHECK for one MAJOR FEATURE: Piecewise function's graphs should NOT CROSS EACH OTHER!

Example 1: Evaluate each piecewise function. Validate points with the function's given graph.

Given Piecewise Function	Graph of Piecewise Function	Evaluate/Complete Table of Values																		
<p>a.)</p> $f(x) = \begin{cases} x^2 + 2x - 3 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$		<table> <tr> <th>x</th><th>Work to find f(x) or y</th><th>Pt (x,y)</th></tr> <tr> <td>-4</td><td>$(-4)^2 + 2(-4) - 3 = 5$</td><td>$(-4, 5)$</td></tr> <tr> <td>-2</td><td>$(-2)^2 + 2(-2) - 3 = -3$</td><td>$(-2, -3)$</td></tr> <tr> <td>0</td><td>$0 + 1 = 1$</td><td>$(0, 1)$</td></tr> <tr> <td>1</td><td>$1 + 1 = 2$</td><td>$(1, 2)$</td></tr> </table>	x	Work to find f(x) or y	Pt (x,y)	-4	$(-4)^2 + 2(-4) - 3 = 5$	$(-4, 5)$	-2	$(-2)^2 + 2(-2) - 3 = -3$	$(-2, -3)$	0	$0 + 1 = 1$	$(0, 1)$	1	$1 + 1 = 2$	$(1, 2)$			
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<p>b.)</p> $f(x) = \begin{cases} -1 & \text{if } x < -3 \\ - x + 5 & \text{if } -3 \leq x < 3 \cup x \neq 0 \\ 3x - 13 & \text{if } x \geq 3 \end{cases}$		<table> <tr> <th>x</th><th>Work to find f(x) or y</th><th>Pt (x,y)</th></tr> <tr> <td>4</td><td>$3(4) - 13 = -1$</td><td>$(4, -1)$</td></tr> <tr> <td>-3</td><td>$- -3 + 5 = 2$</td><td>$(-3, 2)$</td></tr> <tr> <td>-5</td><td>-1</td><td>$(-5, -1)$</td></tr> <tr> <td>3</td><td>$3(3) - 13 = -4$</td><td>$(3, -4)$</td></tr> <tr> <td>0</td><td>undefined</td><td>\emptyset</td></tr> </table>	x	Work to find f(x) or y	Pt (x,y)	4	$3(4) - 13 = -1$	$(4, -1)$	-3	$- -3 + 5 = 2$	$(-3, 2)$	-5	-1	$(-5, -1)$	3	$3(3) - 13 = -4$	$(3, -4)$	0	undefined	\emptyset
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0	undefined	\emptyset																		

Example 2: Make a table of domain values ("work") and graph each piecewise function.

a.)

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \geq -1 \end{cases}$$

(x < -1) (x ≥ -1)

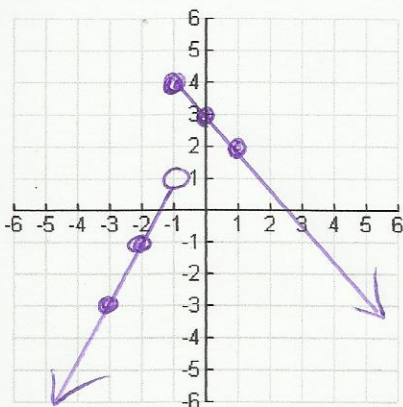


Table of Values	
$2x + 3$	$3 - x$
$(-3, -3)$	$(-1, 4)$
$(-2, -1)$	$(0, 3)$
$(-1, 1)$	$(1, 2)$

b.)

$$f(x) = \begin{cases} \frac{1}{2}|x| - 4 & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$$

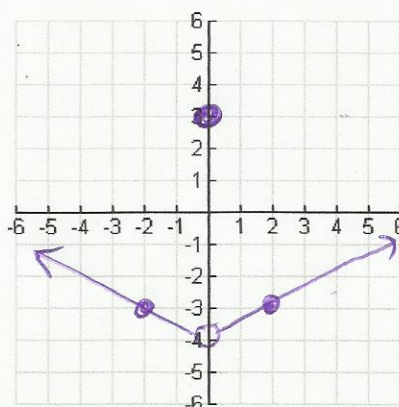


Table of Values	
$\frac{1}{2} x - 4$	3
$(-4, -4)$	$(0, 3)$
$(-2, -3)$	
$(2, -3)$	

Example 2 Cont'd: Make a table of domain values ("work") and graph each piecewise function.

c.) $f(x) = \begin{cases} 3 & \text{if } x \leq -2 \\ 2x-1 & \text{if } -2 < x \leq 1 \\ 4-x & \text{if } x > 1 \end{cases}$ ($x \leq -2$) ($-2 < x \leq 1$) ($x > 1$)

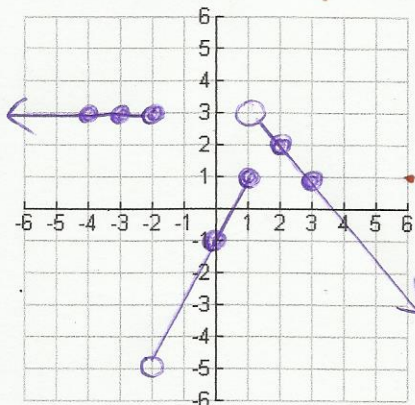


Table of Values		
3	2x-1	4-x
(-2, 3)	(-2, -5)	(1, 3)
(-3, 3)	(0, -1)	(2, 2)
(-4, 3)	(1, 1)	(3, 1)

d.) $f(x) = \begin{cases} -(x+2)^2 + 4 & \text{if } x < -1 \cup x \neq -2 \\ 1 & \text{if } x = -2 \\ 2\sqrt{x+1} - 2 & \text{if } x > -1 \end{cases}$ ($x < -1$) ($x \neq -2$) ($x = -2$) ($x > -1$)

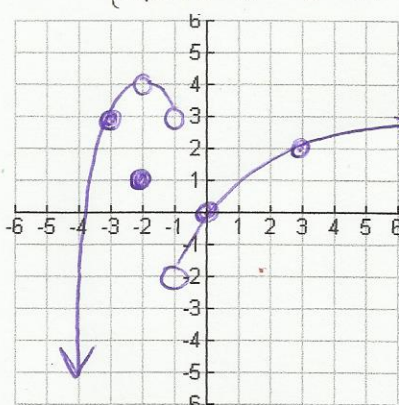


Table of Values		
$-(x+2)^2+4$	1	$2\sqrt{x+1}-2$
(-1, 3)	(-2, 1)	(-1, 2)
(-2, 4)		(0, 0)
(-3, 3)		(1, 2)

Example 3: Complete the problem.

During a particular year, the taxes owed by a married person filing separately with an adjusted gross income of x dollars is given by the piecewise function below:

$$T(x) = \begin{cases} 0.15x & \text{if } 0 \leq x < 17,900 \\ 0.28(x - 17,900) + 2685 & \text{if } 17,900 \leq x < 43,250 \\ 0.31(x - 43,250) + 9783 & \text{if } x \geq 43,250 \end{cases}$$

Find and interpret: $T(70,000) + T(40,000)$

$$T(70,000) = .31(70,000 - 43,250) + 9783 = 18,075.50$$

$$T(40,000) = .28(40,000 - 17,900) + 2685 = 8873$$

$18,075.50 + 8873 = \$26,948.50$
represents how much taxed a married couple owe

Example 4: Determine the domain and range of each piecewise graph in interval notation.

Example 4a	Example 4b	Example 4c	Example 4d
D: $(-\infty, \infty)$	D: $(-\infty, -2) \cup (-2, \infty)$	D: $(-\infty, -3] \cup (-2, 2) \cup [3, \infty)$	D: $(-\infty, -2) \cup [-1, 0) \cup (0, \infty)$
R: $(-3, \infty)$	R: $[-2] \cup (-1, \infty)$	R: $[-4, 4) \cup [5]$	R: $(-\infty, 5)$