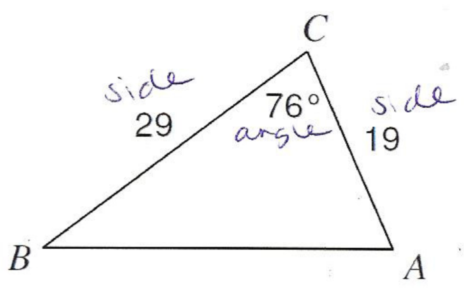
Triangle Trigonometry – Law of Cosines

Before Starting – Think About This!: Let's look at the following problem:



What "type" of triangle is given? SSA AAS ASA SAS SSS

Question: Can you find side c using the Law of Sines?

Answer: No you can not

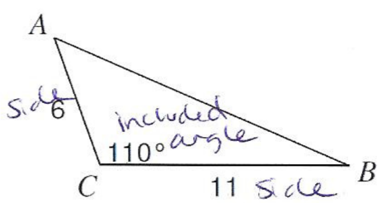
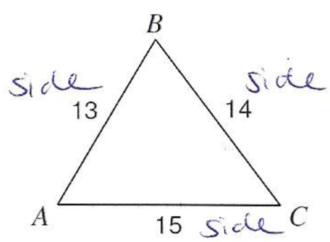
Explanation: ① not enough info ② SAS △ → Use law of cosines
(not SSA, AAS, ASA △)

Law of Cosines "Formulas" → 1.) $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

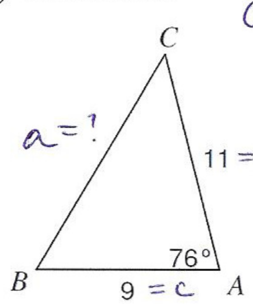
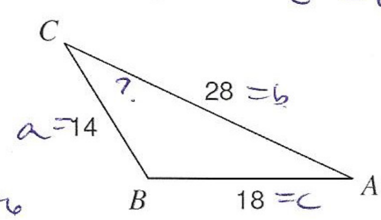
2.) $b^2 = a^2 + c^2 - 2ac \cdot \cos B$

3.) $c^2 = a^2 + b^2 - 2ab \cdot \cos C$

- Only use the Law of Cosines to solve a triangle if given the following:

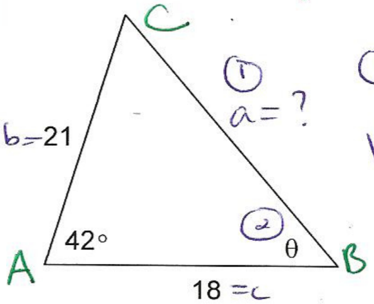
Given Triangle # 1	Given Triangle # 2	Important when Solving w/ LOC
 Type of Triangle: SAS	 Type of Triangle: SSS	• Given SAS triangle → FIRST find… the missing side of given angle • Given SSS triangle → FIRST find… the largest angle which will be across from longest side • Once used the Law of Cosines – May use Law of Sines because will able to set up a proportion

Example 1: Use the Law of Cosines to find the missing angle or side. Round to tenth place.

a.) Find side a.	b.) Find angle C.
$a^2 = b^2 + c^2 - 2bc\cos A$  (for sides do in one BIG step) $a^2 = 11^2 + 9^2 - 2(11)(9)\cos 76$ $a^2 = 154.0994647$ $\sqrt{a^2} = \sqrt{154.0994647}$ $\boxed{a = 12.4}$	$c^2 = a^2 + b^2 - 2ab\cos C$  (for angles do in small "chunks") $18^2 = 14^2 + 28^2 - 2(14)(28)\cos C$ $324 = 980 - 784\cos C$ $-980 \quad -980$ $\frac{-656}{-784} = \frac{-784\cos C}{-784}$ $C = \cos^{-1}\left(\frac{-656}{-784}\right) \rightarrow \boxed{C = 33.2°}$

Example 2: Complete each problem. Round to tenth place.

a.) Find angle θ. (angle B)



① $a^2 = 21^2 + 18^2 - 2(21)(18)\cos 42$

$\sqrt{a^2} = \sqrt{203.1825119}$

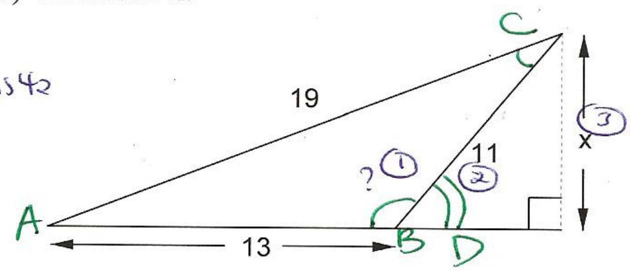
$a = 14.3$

② $\frac{14.3}{\sin 42} = \frac{21}{\sin\theta}$

$\frac{\cancel{14.3}\sin\theta}{\cancel{14.3}} = \frac{21\sin 42}{14.3}$

$\theta = \sin^{-1}\left(\frac{21\sin 42}{14.3}\right) \rightarrow \boxed{\theta = 79.3^\circ}$

b.) Find side x.



① $19^2 = 13^2 + 11^2 - 2(13)(11)\cos B$

$361 = 290 - 286\cos B$

$-290 \qquad -290$

$\frac{71}{-286} = \frac{-286\cos B}{-286}$

$B = \cos^{-1}\left(\frac{71}{-286}\right)$

$B = 104.4^\circ$

② $D = 180 - 104.4$

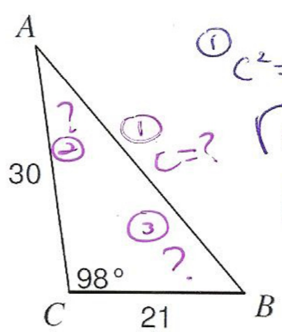
$D = 75.6^\circ$

③ $\sin 75.6 = \frac{x}{11}$

$x = 11\sin 75.6$

$\boxed{x = 10.7}$

c.) Solve triangle ABC: (3 answers)



① $c^2 = 21^2 + 30^2 - 2(21)(30)\cos 98$

$\sqrt{c^2} = \sqrt{1516.358107}$

$\boxed{c = 38.9}$

② $\frac{38.9}{\sin 98} = \frac{21}{\sin A}$

$\frac{\cancel{38.9}\sin A}{\cancel{38.9}} = \frac{21\sin 98}{38.9}$

$A = \sin^{-1}\left(\frac{21\sin 98}{38.9}\right)$

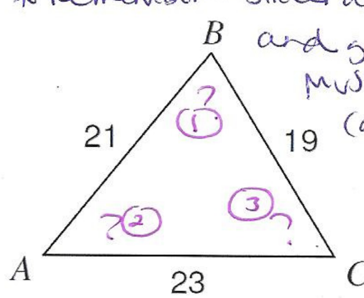
$\boxed{A = 32.3}$

③ $B = 180 - 98 - 32.3$

$\boxed{B = 49.7^\circ}$

d.) Solve triangle ABC: (3 answers)

* Remember – since asked to solve Δ and given SSS → Must find largest angle 1st (across from longest side)



① $23^2 = 19^2 + 21^2 - 2(19)(21)\cos B$

$529 = 802 - 798\cos B$

$\frac{-273}{-798} = \frac{-798\cos B}{-798}$

$B = \cos^{-1}\left(\frac{-273}{-798}\right) \rightarrow \boxed{B = 70^\circ}$

② $\frac{23}{\sin 70} = \frac{19}{\sin A}$

$\frac{23\sin A}{23} = \frac{19\sin 70}{23}$

$A = \sin^{-1}\left(\frac{19\sin 70}{23}\right)$

$\boxed{A = 50.9^\circ}$

③ $C = 180 - 70 - 50.9$

$\boxed{C = 59.1^\circ}$