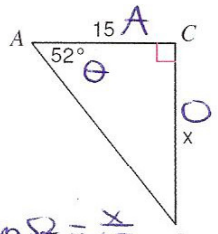
Triangle Trigonometry – Inverse Trigonometric Ratios in Rt Triangles

Before Getting Started – Let's Review: Trigonometric Ratios (x is on top → multiply; x is on bottom → divide by it)

1.) Find side x.

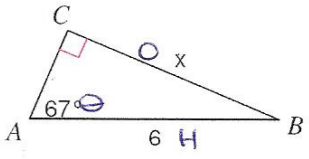


$\tan 52 = \frac{x}{15}$

$x = 15 \tan 52$

$x = 19.2$

2.) Find side x.

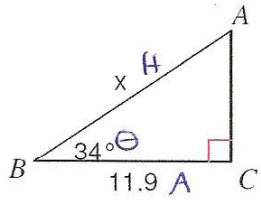


$\sin 67 = \frac{x}{6}$

$x = 6 \sin 67$

$x = 5.5$

3.) Find side x.

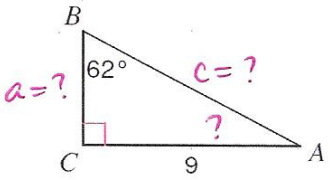


$\cos 34 = \frac{11.9}{x}$

$x = \frac{11.9}{\cos 34}$

$x = 14.4$

4.) Solve triangle ABC. (3 answers)



$A = 90 - 62$

$A = 28°$

$\tan 62 = \frac{9}{a}$

$a = \frac{9}{\tan 62}$

$a = 4.8$

$4.8^2 + 9^2 = c^2$

$c^2 = 104.04$

$c = 10.2$

Inverse Trigonometric Ratios: Use ONLY when FINDING AN ANGLE MEASUREMENT!!!

Trigonometric Ratios	Inverse Trigonometric Ratios	Calculator Keys
$\sin\theta = \frac{opposite}{hypotenuse}$	$\theta = \sin^{-1}\left(\frac{O}{H}\right)$	2nd sin ($\sin^{-1}$) (arc sin)
$\cos\theta = \frac{adjacent}{hypotenuse}$	$\theta = \cos^{-1}\left(\frac{A}{H}\right)$	2nd cos ($\cos^{-1}$) (arc cos)
$\tan\theta = \frac{opposite}{adjacent}$	$\theta = \tan^{-1}\left(\frac{O}{A}\right)$	2nd tan ($\tan^{-1}$) (arc tan)

Example 1: Solve each equation by find the value of angle θ. Round to tenth place.

a.) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = 60°$

b.) $\cos\theta = 0.7498$

$\theta = \cos^{-1}(.7498)$

$\theta = 41.4°$

c.) $\tan\theta = 2$

$\theta = \tan^{-1}(2)$

$\theta = 63.4°$

Example 2: Evaluate each expression using the inverse trig ratios' definitions.

a.) $\tan\left(\tan^{-1}\frac{6}{4}\right)$

(inverses of each other)

$= \frac{6}{4}$

$= \frac{3}{2}$

b.) cos (arcsin 1)

$= \cos(\sin^{-1} 1)$

$= \cos(90°)$

$= 0$

c.) $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

$= \sin(30°)$

$= \frac{1}{2}$

d.) tan (arccos 0)

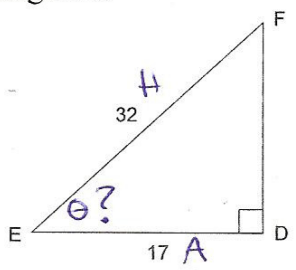
$= \tan(\cos^{-1} 0)$

$= \tan(90)$

$= \emptyset$ (undefined)

Example 3: **Find the indicated angle. Round to nearest tenth.**

a.) Find angle E..

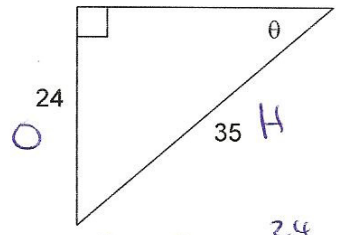


$\cos E = \frac{17}{32}$

$E = \cos^{-1}\left(\frac{17}{32}\right)$

$E = 57.9°$

b.) Find angle θ.

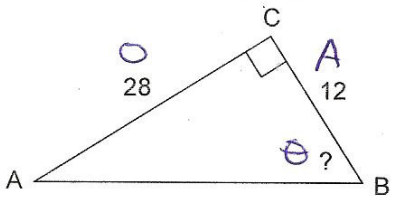


$\sin\theta = \frac{24}{35}$

$\theta = \sin^{-1}\left(\frac{24}{35}\right)$

$\theta = 43.3°$

c.) Find the missing angle.

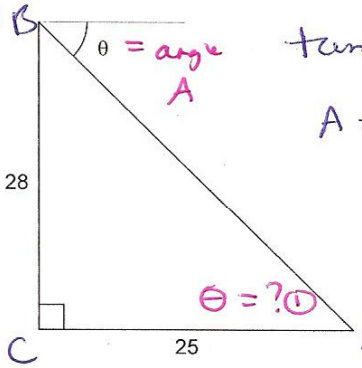


$\tan B = \frac{28}{12}$

$B = \tan^{-1}\left(\frac{28}{12}\right)$

$B = 66.8°$

d.)



$\tan A = \frac{28}{25}$

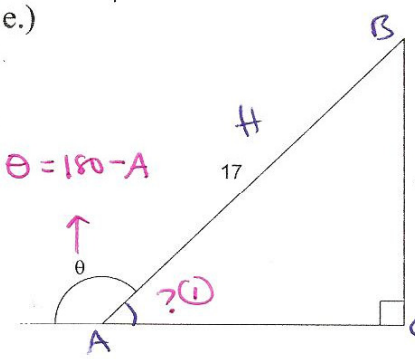
$A = \tan^{-1}\left(\frac{28}{25}\right)$

$A = 48.2$

So $\theta = 48.2°$

(alternate interior angles)

e.)



$\sin A = \frac{10}{17}$

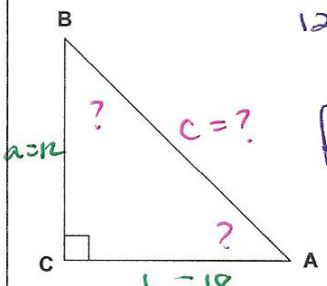
$A = \sin^{-1}\left(\frac{10}{17}\right)$

$A = 36°$

$\theta = 180 - 36$

$\theta = 144°$

f.) Solve triangle ABC: a = 12 , b = 18



$12^2 + 18^2 = c^2$

$c^2 = 468$

$c = 21.6$

$\tan A = \frac{12}{18}$

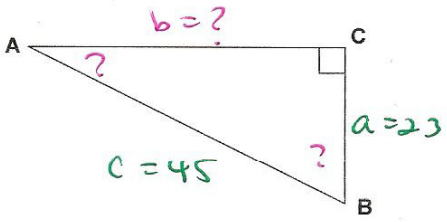
$A = \tan^{-1}\left(\frac{12}{18}\right)$

$A = 33.7°$

$B = 90 - 33.7$

$B = 56.3°$

g.) Solve triangle ABC: a = 23, c = 45



$23^2 + b^2 = 45^2$

$-23^2 \quad -23^2$

$b^2 = 1496$

$b = 38.7$

$\sin A = \frac{23}{45}$

$A = \sin^{-1}\left(\frac{23}{45}\right)$

$A = 30.7°$

$B = 90 - 30.7 \rightarrow B = 59.3°$