# Angles, Unit Circle, Trig Graphs/Equations – Graphing S/C Functions

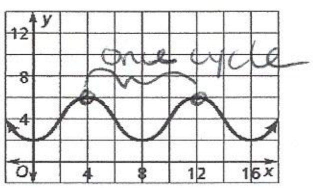
## Periodic Function and Period

- One basic property of both the sine and the cosine function → considered periodic

– **periodic function** → a function that repeats a pattern of y-values at regular intervals where one cycle = one period.

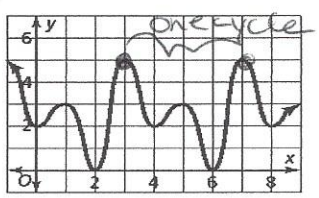
– **period (of a periodic function)** → the horizontal length of one cycle

**Example 1: Determine if the given function is periodic. If so, state the period.**

1.)


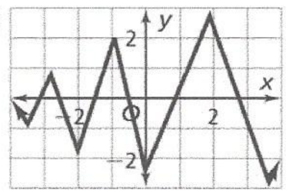

Periodic? Yes No

Period = 8

2.)


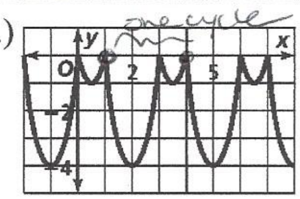

Periodic? Yes No

Period = 4

3.)


Periodic? Yes No

Period = N/A (none)

4.)



Periodic? Yes No

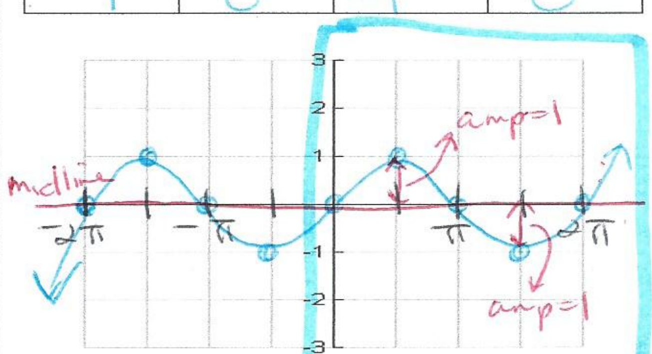
Period = 3

## Graph of Trig Function # 1 - Sine

Make a table of domain values between ± $2\pi$ for the function of $y = \sin(x)$

| $-2\pi$ | $-\frac{3\pi}{2}$ | $-\pi$ | $-\frac{\pi}{2}$ | $0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | −1 | 0 |

| $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|
| 1 | 0 | −1 | 0 |




**Characteristics of the Sine Function:**

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$

Period: $2\pi$ Amplitude: 1

(4th dash)

Important Part of Graph: Domain is Positive

1.) Start (0,0) 4.) down

2.) up 5.) up

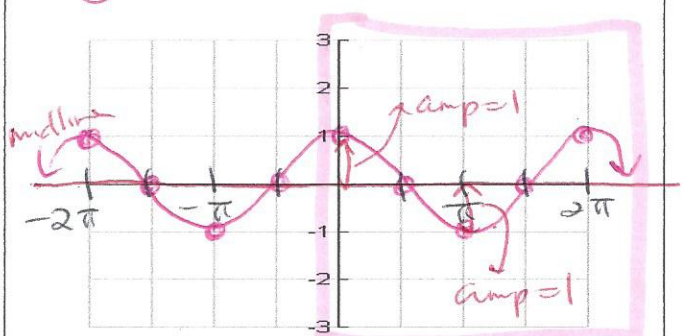
3.) down looks like a sideway S

## Graph of Trig Function # 2 - Cosine

Make a table of domain values between ± $2\pi$ for the function of $y = \cos(x)$

| $-2\pi$ | $-\frac{3\pi}{2}$ | $-\pi$ | $-\frac{\pi}{2}$ | $0$ |
|---|---|---|---|---|
| 1 | 0 | −1 | 0 | 1 |

| $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|
| 0 | −1 | 0 | 1 |




**Characteristics of the Cosine Function:**

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$

Period: $2\pi$ Amplitude: 1

(4th dash)

Important Part of Graph: Domain is Positive

1.) Start (0,1) 4.) up

2.) down 5.) up

3.) down looks like a U (parabola)

**Graphing the Sine / Cosine Function:** $y = f(x) = a\sin(bx \pm c) \pm d$ or $y = f(x) = a\cos(bx \pm c) \pm d$

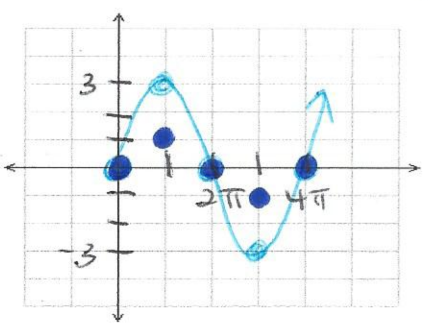
- Each parameter (letter) affects the graph of $y = a \sin/\cos(bx \pm c) \pm d$ differently: * amp does not affect things at 0 *
  - Parameter __a__ affects the __range__ of f (x) where __$|a|$__ is called the __amplitude__
  - Parameter __b__ affects the __period__ of f (x) where the __period $= \frac{2\pi}{b}$__
  - Parameter __c__ affects the __horizontal (phase) shift__ of f (x) where __phase shift $= -\frac{c}{b}$__
    - If __$-\frac{c}{b} < 0$__ then the graph shifts to the __left__ - If __$-\frac{c}{b} > 0$__ then the graph shifts to the __right__
  - Parameter __d__ affects the __vertical shift__ of f (x)
    - If __$d < 0$__ then the graph shifts __down__ - If __$d > 0$__ then the graph shifts __up__

**Example 2: State the amplitude, period, and phase shift of each function.**

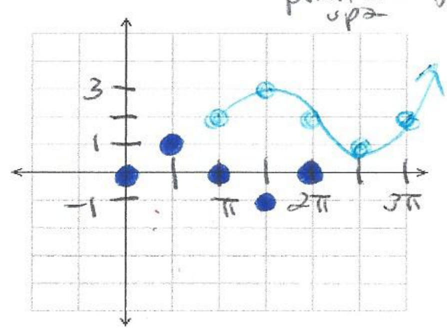
| Function | Amplitude | Period | Phase Shift | Vertical Shift |
|---|---|---|---|---|
| a.) $y = 3\sin(2x) + 1$ (a, b, c=0, d) | $\lvert 3\rvert = \boxed{3}$ | $\frac{2\pi}{2} = \boxed{\pi}$ | none | up 1 |
| b.) $y = -2\cos\left(1x + \frac{\pi}{2}\right)$ (a, b, c, d=0) | $\lvert -2\rvert = \boxed{2}$ | $\frac{2\pi}{1} = \boxed{2\pi}$ | $\frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2} = \boxed{\text{left } \frac{\pi}{2}}$ | none |
| c.) $y = 1\sin(4x - \pi) - 3$ (a, b, c, d) | $\lvert 1\rvert = \boxed{1}$ | $\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ | $\frac{\pi}{4} = \boxed{\text{right } \frac{\pi}{4}}$ | down 3 |
| d.) $y = \frac{1}{2}\cos\left(\frac{1}{4}x + \pi\right) + 2$ (a, b, c, d) | $\lvert \frac{1}{2}\rvert = \boxed{\frac{1}{2}}$ | $\frac{2\pi}{\frac{1}{4}} = \boxed{8\pi}$ | $\frac{-\pi}{\frac{1}{4}} = -4\pi\ \boxed{\text{left } 4\pi}$ | up 2 |

**Example 3: Graph each function by finding the amplitude, period, phase shift, and vertical shift.**

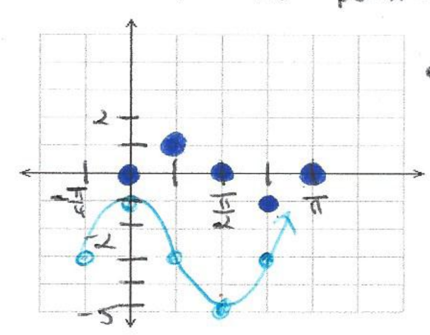
a.) $y = 3\sin(\frac{1}{2}x)$ amp = 3, period $= \frac{2\pi}{1/2} = 4\pi$



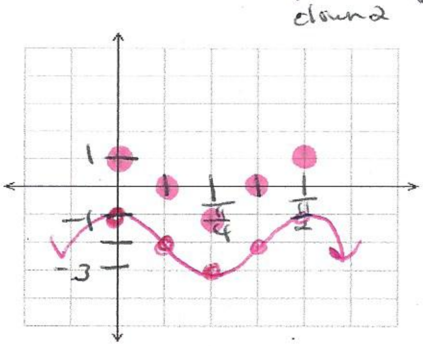

b.) $y = \sin(x - \pi) + 2$ amp = 1, period = $2\pi$, pshift = $\pi$ right, up 2



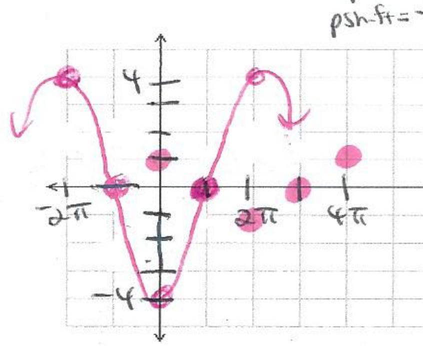

c.) $y = 2\sin\left(2x + \frac{\pi}{2}\right) - 3$ amp = 2, period $= \frac{2\pi}{2} = \pi$, pshift $= \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$ left, down 3




d.) $y = \cos(4x) - 2$ amp = 1, period $= \frac{2\pi}{4} = \frac{\pi}{2}$, down 2




e) $y = 4\cos(\frac{1}{2}x + \pi)$ amp = 4, period $= \frac{2\pi}{1/2} = 4\pi$, pshift $= \frac{-\pi}{1/2} = -2\pi$ left




f.) $y = 3\cos\left(x - \frac{\pi}{2}\right) + 1$ amp = 3, period = $2\pi$, pshift $= \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$ right, up 1