Geometric WP WS

1) $a_1 = 24 \quad r = .6\ (60\%)$

$n = 6$ ("after" fifth bounce)

$a_6 = 24(.6)^5$

$a_6 = 1.87$

→ **1.87 ft**

2) $a_1 = 5$

$a_2 = 5.50$

$a_3 = 6.05$

$r = \frac{5.50}{5} = 1.1$

$n = 52$ (wks in year)

$a_{52} = 5(1.1)^{51}$

$a_{52} = 645.4497$

→ **$645.65**

3) $a_1 = 5 \quad r = 5 \quad n = 7$ (# days in wk)

$S_7 = \frac{5(1-5^7)}{(1-5)} = 97655$

→ **97,655 people**

4) $a_1 = 200 \quad r = 2$ (doubles)

$n = 7$ (for 12 hrs)

($n=1$; $n=2 \to 2$ hrs ; $n=3 \to 4$ hrs ; $n=4 \to 6$ hrs ; $n=5 \to 8$ hrs ; $n=6 \to 10$ hrs)

$a_7 = 200(2)^6$

$a_7 = 12,800$

→ **12,800 bacterium**

5) $a_4 = 10.62$ (for 1983)

$a_8 = 11.92$ (for 1987)

$a_{21} = ?$ (for 2000)

$\begin{cases} 11.92 = a_1 r^7 \\ 10.62 = a_1 r^3 \end{cases}$

$\frac{10.62}{(1.029)^3} = \frac{a_1 (1.029)^3}{(1.029)^3}$

$1.122 = r^4$

$\sqrt[4]{1.122} = \sqrt[4]{r^4}$

$r = 1.029$

$a_1 = 9.746$

$a_{21} = 9.746(1.029)^{20}$

$a_{21} = 17.3$

→ **17.3 million people**

6) $a_0 = 40,000$

$a_1 = 41,600$

$r = \frac{41600}{40000}$

$r = 1.04$

$a_5 = ?$ (4 more raises)

$a_5 = 4160(1.04)^4$

$a_5 = 48666.1161$

→ **$48,666.12**

7) $a_1 = 8$

$S_n = 15.75$

$r = .5$

$15.75 = \frac{8(1-.5^n)}{(1-.5)}$

$15.75 = \frac{8(1-.5^n)}{.5}$

$\frac{15.75}{16} = \frac{16(1-.5^n)}{16}$

$.984375 = 1 - .5^n$

$-1 \quad -1$

$-.015625 = -.5^n$

$\frac{\log .015625}{\log .5} = \frac{n \log .5}{\log .5}$

$n = 6$

6 swings

8) $a_1 = 27$

$a_4 = 8$

$r = ?$

$\frac{8}{27} = \frac{27(r)^3}{27}$

$\frac{8}{27} = r^3$

$\sqrt[3]{\frac{8}{27}} = \sqrt[3]{r^3}$

$\boxed{r = \frac{2}{3} \text{ or } 66.7\%}$

9) $r = 1.05$ (raise is after 1st year)

$S_{40} = 3624000$

$a_1 = ?$

$3624000 = \frac{a_1(1-1.05^{40})}{(1-1.05)}$

$\frac{3624000}{120.7997742} = \frac{a_1 \cdot 120.7997742}{120.7997742}$

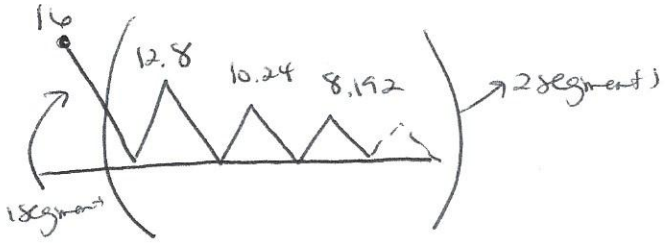
$a_1 = 30{,}000.05608$

$\approx \boxed{\$30{,}000}$

10) $a_1 = 16(.8) = 12.8$

$r = .8$ (for 80%) $n = 15$

$S_{15+16} = ?$



$\Rightarrow 16 + 2\left(\frac{12.8(1-.8^{15})}{(1-.8)}\right)$

$= \boxed{139.5 \text{ ft}}$

11) $a_1 = 8 \quad r = \frac{4}{5} \quad S = ?$

$S = \frac{8}{(1-(4/5))} = 40$

$\rightarrow \boxed{40 \text{ ft}}$

12) $a_1 = 90 \quad S = 900 \quad r = ?$

$\frac{900}{1} * \frac{90}{1-r}$

$900 - 900r = 90$

$-900 \qquad -900$

$\frac{-900r}{-900} = \frac{-810}{-900}$

$r = .9$

$\rightarrow \boxed{90\%}$

13) $a_1 = 24$

$S = 72$

$a_3 = ?$

$72 = \frac{24}{1-r}$

$72 - 72r = 24$

$-72 \qquad -72$

$\frac{-72r}{-72} = \frac{-48}{-72}$

$r = \frac{2}{3}$

$a_3 = 24\left(\frac{2}{3}\right)^2$

$\approx \boxed{10.7 \text{ cm}}$

14) $0.672727272\ldots$

$= .6 + [.072 + .00072 + .0000072 + \ldots]$

$= .6 + \left(\frac{.072}{1-.01}\right)$

$= \frac{3}{5} + \frac{4}{55} = \frac{37}{55}$

Susan is correct. She forgot to add in the part that wasn't a series (.6)

15) $S = 30$

$r = .7$ (for 70%)

$a_1 = ?$

$30 = \frac{a_1}{1-.7}$

$.3 \times 30 = \frac{a_1}{.3} \times .3$

$a_1 = 9 \rightarrow$ 9 ft

16) a) $6 + 4 + \frac{8}{3} + \cdots$

$r = \frac{4}{6} = \frac{2}{3}$ ✓

$\Rightarrow S = \frac{6}{(1-(2/3))} \Rightarrow$ converges to 18

b) $4 - 8 + 16 - \cdots$

$r = \frac{-8}{4} = -2$ ✗

$\Rightarrow S = DNE \Rightarrow$ diverges

c) $-98 - 73.5 - 55.125 - \cdots$

$r = \frac{-73.5}{-98} = .75$ ✓

$\Rightarrow S = \frac{-98}{(1-.75)} \Rightarrow$ converges to -392

d) $\sum_{n=1}^{\infty} 4\left(\frac{1}{5}\right)^{n-1}$ ($a_1 = 4$, $r = \frac{1}{5}$) ✓

$\Rightarrow S = \frac{4}{(1-(1/5))} \Rightarrow$ converges to 5

e) $\sum_{n=1}^{\infty} 2(4)^{n-1}$ ($a_1 = 2$, $r = 4$ ✗)

$\Rightarrow S = DNE \Rightarrow$ diverges

f) $\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{5}{4}\right)^{n-1}$ ($a_1 = \frac{1}{2}$, $r = \frac{5}{4} = 1.25$)

$\Rightarrow S = DNE \Rightarrow$ diverges