Sequences and Series – Geometric Sequences

Specific Sequence # 2 – Geometric Sequence

Before Getting Started – The following are examples of geometric sequences:

1.) 12 , 96 , 768 , 6144 , 49152 , 393216 Pattern Rule: multiply by 8

2.) $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}$, $\frac{1}{32}$, $-\frac{1}{64}$ Pattern Rule: multiply by $-\frac{1}{2}$ or divide by -2

What do you think happens in geometric sequences? keep multiply/divide by same #

- **geometric sequence** → a sequence where the ratio between any two consecutive terms is a constant, called r, the common ratio

Example 1: Determine if these sequences are geometric. If so, state the common ratio.

a.) 3 , 9 , 27 , 81 , ...
$\frac{9}{3}=3$ $\frac{27}{9}=3$ $\frac{81}{27}=3$ Yes → r = 3

b.) 8 , 14 , 24.5 , 36.75 , ...
$\frac{14}{8}=1.75$ $\frac{24.5}{14}=1.75$ $\frac{36.75}{24.5}=1.5$ NO → r = N/A

c.) 96 , −24 , 6 , $-\frac{3}{2}$
$\frac{-24}{96}=-.25$ $\frac{6}{-24}=-.25$ $\frac{-3/2}{6}=-.25$ Yes → r = −.25

Example 2: Find the next three terms of each geometric sequence. Write your answer as sequence.

a.) $a_1 = 4$ and $r = -5$
$a_1 = 4$
$a_2 = 4 \cdot -5 = -20$
$a_3 = -20 \cdot -5 = 100$
$a_4 = 100 \cdot -5 = -500$
4, −20, 100, −500, ...

b.) 1134 , 378 , 126 , ...
$r = \frac{378}{1134}$
$r = \frac{1}{3}$
$a_4 = 126 \div 3 = 42$
$a_5 = 42 \div 3 = 14$
$a_6 = 14 \div 3 = \frac{14}{3}$
1134, 378, 126, 42, 14, $\frac{14}{3}$, ...

"Nth Term Formula" of Geometric Sequence: Used to find ANY term of a geometric sequence

Consider a geometric sequence whose first term is a_1 and whose common ratio is r:

a_1 → 1st term (a_1)

$a_1 \cdot r$ → 2nd term ($a_2 = a_1 \cdot r$)

$a_1 \cdot r \cdot r = a_1 \cdot r^2$ → 3rd term ($a_3 = a_1 \cdot r^2$)

$a_1 \cdot r^2 \cdot r = a_1 r^3$ → 4th term ($a_4 = a_1 \cdot r^3$)

$a_1 \cdot r^3 \cdot r = a_1 r^4$ → 5th term ($a_5 = a_1 \cdot r^4$)

(General) n^th term Formula: $a_n = a_1 (r)^{n-1}$ → some important notes about this formula…

- formula will always be a exponential equation
- do not multiply a_1 # and r # together to simplify the formula
- put ( ) around any "r" that's a negative # or fractional #

Example 3: Find what is indicated for each geometric sequence. * given as fraction → answers as fractions *

a.) $a_1 = 2$ and $r = 4$, find the 8^{th} term ($n = 8$)

$a_8 = 2(4)^{8-1}$

$a_8 = 2(4)^7$

$\boxed{a_8 = 32768}$

b.) Find a_5 ($n = 5$) for the sequence $-1, \frac{1}{4}, -\frac{1}{16}, \ldots$

a_1 $r = \frac{1}{4} \div -1 = -\frac{1}{4}$

$a_5 = -1\left(-\frac{1}{4}\right)^{5-1}$

$a_5 = -1\left(-\frac{1}{4}\right)^4$

$\boxed{a_5 = -\frac{1}{256}}$

c.) Write the nth term formula (equation) for the sequence $3, -36, 432, \ldots$

a_1 $r = \frac{-36}{3} = -12$

$\boxed{a_n = 3(-12)^{n-1}}$

note: can't simplify as $a_n = (-36)^{n-1}$

Example 4: Considering all given sequences are geometric – Find what is asked.

a.) The 7^{th} term of the sequence is 470596 and the common ratio is 7. What is the first term?

$a_7 = 470596 \quad r = 7 \quad a_1 = ?$

$470596 = a_1(7)^{7-1}$

$\frac{470596}{117649} = \frac{a_1 \cdot 117649}{117649}$

$\boxed{a_1 = 4}$

b.) The sixth term of the sequence is -6250 and the first term is -2. What is the common ratio?

$a_6 = -6250 \quad a_1 = -2 \quad r = ?$

$-6250 = -2(r)^{6-1}$

$\frac{-6250}{-2} = \frac{-2r^5}{-2}$

$3125 = r^5$

$\sqrt[5]{3125} = \sqrt[5]{r^5} \rightarrow \boxed{r = 5}$

c.) The third term is 20 and the fifth term is five. What is ninth term?

$a_3 = 20 \quad a_5 = 5 \quad a_9 = ?$

(system of eq b/c geo – bigger term # of top since it's geometric)

$\div \begin{cases} 5 = a_1 r^4 \\ 20 = a_1 \cdot r^2 \end{cases}$

$\frac{1}{4} = 1 \cdot r^2$

$\frac{1}{4} = r^2$

$\sqrt{\frac{1}{4}} = \sqrt{r^2} \rightarrow r = \frac{1}{2}$

$5 = a_1\left(\frac{1}{2}\right)^4$

$\frac{5}{\frac{1}{16}} = \frac{\frac{1}{16} a_1}{\frac{1}{16}}$

$a_1 = 80$

$a_9 = 80\left(\frac{1}{2}\right)^{9-1}$

$\boxed{a_9 = \frac{5}{16}}$

d.) Which term is 78,732 in the sequence of $4, 12, 36, 108, \ldots$?

$a_n = 78732 \quad a_1 = 4 \quad r = \frac{12}{4} = 3 \quad n = ?$

$\frac{78732}{4} = \frac{4(3)^{n-1}}{4}$

$19683 = 3^{n-1}$

$\log(19683) = \log 3^{n-1}$

$1 + \frac{\log(19683)}{\log 3} = \frac{(n-1)\log 3}{\log 3} + 1$

$n = 10 \rightarrow$ 10th term

- **geometric means** → represent the terms between any two nonsuccessive terms of a geometric sequence

Ex: Circle the 3 geometric means between 2 and 162: $2, \textcircled{6}, \textcircled{18}, \textcircled{54}, 162, \ldots$

Example 5: Find the geometric means of the geometric sequence below.

$2.25, \underline{9}, \underline{36}, \underline{144}, 576$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$2.25 \times 4 \quad 9 \times 4 \quad 36 \times 4 \quad 144 \times 4 = 576$ ✓

$\frac{576}{2.25} = \frac{2.25(r)^{5-1}}{2.25}$

$256 = r^4$

$\sqrt[4]{256} = \sqrt[4]{r^4}$

$r = 4$