

Functions/Regression – Exponential and Logarithmic Equations

Solving Exponential Equations Examples:

Notes: 1.) Keep answers as (reduced) fractions when possible.

If a decimal can NOT be turned into a fraction – ROUND to 3 decimal places

Exponential Equations: Type 1 – Both Sides have the Same Base

<p>1a.) $2^{2x+14} = 64$</p> <p>$2^{2x+14} = 2^6$</p> <p>$2x+14 = 6$</p> <p>$-14 -14$</p> <p>$2x = -8$</p> <p>$\frac{2x}{2} = \frac{-8}{2}$</p> <p>$x = -4$</p>	<p>1b.) $4^{3x} = 32^{x+1}$</p> <p>$(2^2)^{3x} = (2^5)^{x+1}$</p> <p>$2 \cdot 3x = 5(x+1)$</p> <p>$6x = 5x + 5$</p> <p>$-5x -5x$</p> <p>$x = 5$</p>	<p>1c.) $3^{2x+5} = \left(\frac{1}{9}\right)^{x-1}$</p> <p>$3^{2x+5} = (3^{-2})^{x-1}$</p> <p>$2x+5 = -2(x-1)$</p> <p>$2x+5 = -2x+2$</p> <p>$+2x +5 +2x -5$</p> <p>$4x = -3$</p> <p>$\frac{4x}{4} = \frac{-3}{4}$</p> <p>$x = -\frac{3}{4}$</p>	<p>1d.) $25 \cdot \left(\frac{1}{125}\right)^{x+4} = \sqrt[5]{3125}$</p> <p>$5 \cdot (5^{-3})^{x+4} = 5^{\frac{5}{2}}$</p> <p>$2 + -3(x+4) = \frac{5}{2}$</p> <p>$2 - 3x - 12 = 2.5$</p> <p>$-3x - 10 = 2.5$</p> <p>$+10 +10$</p> <p>$-3x = 12.5$</p> <p>$\frac{-3x}{-3} = \frac{12.5}{-3}$</p> <p>$x = -\frac{25}{6}$</p>
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Exponential Equations: Type 2: Both Sides are NOT the Same Base – Take log (or ln) of both sides

<p>1e.) $3^{2x-1} = 7$</p> <p>$\log 3^{2x-1} = \log 7$</p> <p>$(2x-1) \log 3 = \log 7$</p> <p>$\frac{(2x-1) \log 3}{\log 3} = \frac{\log 7}{\log 3}$</p> <p>$2x-1 = 1.771244$</p> <p>$+1 +1$</p> <p>$2x = 2.771244$</p> <p>$\frac{2x}{2} = \frac{2.771244}{2}$</p> <p>$x = 1.386$</p>	<p>1f.) $3 \cdot 4^x + 11 = 2$</p> <p>$3 \cdot 4^x = -9$</p> <p>$4^x = -3$</p> <p>$\log 4^x = \log(-3)$</p> <p>$x \log 4 = \log(-3)$</p> <p>$\frac{x \log 4}{\log 4} = \frac{\log(-3)}{\log 4}$</p> <p>can't do</p> <p>no solution</p>	<p>1g.) $2e^{5x-3} = 16$</p> <p>$e^{5x-3} = 8$</p> <p>$\ln e^{5x-3} = \ln(8)$</p> <p>$5x-3 = \ln(8)$</p> <p>$+3 +3$</p> <p>$5x = \ln(8) + 3$</p> <p>$\frac{5x}{5} = \frac{\ln(8) + 3}{5}$</p> <p>$x = 1.016$</p>	<p>1h.) $\frac{12}{1+e^{-x}} = 2$</p> <p>$\frac{2(1+e^{-x})}{2} = \frac{12}{2}$</p> <p>$1+e^{-x} = 6$</p> <p>$-1 -1$</p> <p>$e^{-x} = 5$</p> <p>$\ln e^{-x} = \ln(5)$</p> <p>$-x = \ln(5)$</p> <p>$\frac{-x}{-1} = \frac{\ln(5)}{-1}$</p> <p>$x = -1.609$</p>
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Solving Logarithmic Equations Examples:

Notes: 1.) Keep answers as (reduced) fractions when possible – if not, ROUND to 3 places!

2.) Remember – You CAN NOT take log of a negative number or zero (b/c VA: $x = 0$)

Some solutions MAY OR MAY NOT WORK (could also have no solutions)!

When done solving a log equation – Check to see that it works in the original problem!

Logarithmic Equations: Type 1 – Both sides (and every term) is a logarithm

<p>2a.) $\log_5(6-3x) = \log_5(10-5x)$</p> $\begin{array}{r} 5 \quad 5 \\ 6-3x = 10-5x \\ -6 \quad +5x \quad -6 \quad +5x \\ \hline 2x = 4 \\ \frac{2x}{2} = \frac{4}{2} \\ x = 2 \text{ (will get } \log(0) \text{)} \end{array}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">no solution</div>	<p>2b.) $\log(x-3) + \log x = \log 28$</p> $\begin{array}{l} \log(x-3) \cdot x = \log 28 \\ 10 \quad 10 \\ x^2 - 3x = 28 \\ x^2 - 3x - 28 = 0 \\ (x+4)(x-7) = 0 \\ x+4=0 \quad x-7=0 \\ x=-4 \quad x=7 \\ \text{(will get } \log(\text{neg}^\#) \text{)} \quad \downarrow \text{only answer} \end{array}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">X = 7</div>	<p>2c.) $\ln(2x+1) - \ln(x-1) = \ln 7$</p> $\begin{array}{l} \ln\left(\frac{2x+1}{x-1}\right) = \ln 7 \\ e \quad e \\ \frac{2x+1}{x-1} = \frac{7}{1} \\ 7x-7 = 2x+1 \\ -2x \quad +7 \quad -2x \quad +7 \\ \hline 5x = 8 \\ \frac{5x}{5} = \frac{8}{5} \\ \boxed{X = \frac{8}{5}} \end{array}$
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Logarithmic Equations: Type 2 – SINGLE LOG on one side and a CONSTANT on the other side

<p>2d.) $\log_3(2x+15) = 2$</p> $\begin{array}{l} 3^2 = 2x+15 \\ 9 = 2x+15 \\ -15 \quad -15 \\ \hline -6 = 2x \\ \frac{-6}{2} = \frac{2x}{2} \\ \boxed{x = -3} \end{array}$	<p>2e.) $\log_2 x + \log_2(x-2) = 3$</p> $\begin{array}{l} \log_2 x(x-2) = 3 \\ 2^3 = x^2 - 2x \\ 8 = x^2 - 2x \\ x^2 - 2x - 8 = 0 \\ (x+2)(x-4) = 0 \\ x+2=0 \quad x-4=0 \\ x=-2 \quad x=4 \\ \text{(will get } \log(\text{neg}^\#) \text{)} \quad \downarrow \text{only answer} \end{array}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">X = 4</div>	<p>2f.) $4 - 6\ln(3x-1) = -20$</p> $\begin{array}{l} -4 \quad -4 \\ -6\ln(3x-1) = -24 \\ -6 \quad -6 \\ \hline \ln(3x-1) = 4 \\ e \quad e \\ 3x-1 = e^4 \\ +1 \quad +1 \\ \hline 3x = (e^4 + 1) \\ \frac{3x}{3} = \frac{(e^4 + 1)}{3} \\ \boxed{X = 18.533} \end{array}$
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