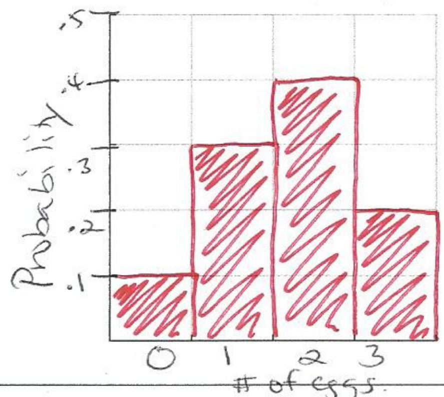
Advanced Functions – Discrete Random Variables WS Name: Key

Directions: Read each problem and answer the questions that follow either on the worksheet paper or on a separate sheet (just make sure all questions are answered). Construct complete and neat histograms and tables.

1.) Parus caeruleus is a small, blue, and yellow bird common to Great Britain that always lays three eggs in its nest. The number of eggs, x, which actually hatch has the following probability distribution.

x = number of hatched eggs	0	1	2	3
Probability	0.1	0.3	0.4	0.2

a.) Make a probability histogram.



b.) If a nest is selected at random, how many eggs would we expect to hatch?

$0(.1)+1(.3)+2(.4)+3(.2) = \boxed{1.7 \text{ eggs}}$

c.) What percent of nests have 1 or less eggs hatch?

$.1+.3 = .4 = \boxed{40\%}$

d.) What percent of nests have at least 2 eggs hatch?

$.4+.2 = .6 = \boxed{60\%}$

2.) A small airport is interested in the number of late aircraft arrivals per day. Thus, everyday for a year, managers count the daily number of late arrivals.

x = number of late arrivals	0	1	2	3	4 +
Probability	0.118	0.186	.296	0.260	0.140

a.) What is the probability of the airport having 2 late arrivals a day?

$.118+.186+.260+.140 = .704 \quad 1-.704 = \boxed{.296}$

b.) What percent will the airport have 3 or more late arrivals?

$.260+.140 = .40 = \boxed{40\%}$

c.) How often will there be 1 or less late arrivals?

$.118+.186 = .304 = \boxed{30.4\%}$

d.) On any particular day, how many late arrivals would we expect?

$0(.118)+1(.186)+2(.296)+3(.260)+4(.140) = \boxed{2.118 \text{ late arrivals}}$

3.) In 1952, Dr. Virginia Apgar suggested five criteria for measuring a baby's health at birth. Still used today, each category — skin color, heart rate, muscle tone, breathing, and response to stimulation — receives a 0, 1, or 2. Thus, a newborn's *Apgar score* will be an integer between 0 and 10. The table displays the probabilities for each possible Apgar score.

x = Apgar Score	0	1	2	3	4	5	6	7	8	9	10
Probability	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.437	0.053

a.) A score of 7 or more indicates a healthy baby: what percentage of newborns are considered "healthy"?

$.099+.319+.437+.053 = .908 = \boxed{90.8\%}$

b.) What is the expected Apgar score for a randomly selected newborn? Is a typical newborn healthy?

$0(.001)+1(.006)+2(.007)+3(.008)+4(.012)+5(.020)$
$+6(.038)+7(.099)+8(.319)+9(.437)+10(.053) = \boxed{8.128}$

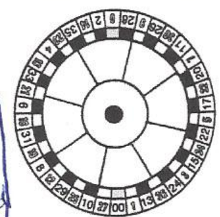
$\boxed{\text{Yes since } 8.128 > 7}$

4.) The American casino game, Roulette, features a recumbent wheel with 38 "slots" along its circumference. Eighteen slots are red, eighteen are black, and two slots – numbered 0 and 00 – are green. As the wheel spins, a marble is tossed onto it, where it will ultimately land in one of the slots. When a player bets $1 on red, and the marble lands on red, the players get the dollar back and another $1 for "winning." Of course, if a player bets on red and the marble lands on another color, the player loses $1. Thus, since there are only two possible values, we can easily define x as a player's net gain: either +$1 or –$1.

a.) Make a table showing the possible values for x and their associated probabilities.

	Not Red	Red
x	−1	1
Probability	$\frac{18}{38}$	$\frac{16}{38}$

b.) Find the expected value for this probability distribution.
What does this say about a player's chances of winning in the long run?



$-1\left(\frac{18}{38}\right)+1\left(\frac{16}{38}\right)=-.0526=$ $\boxed{\text{player will lose about 5 cents for each game played}}$

5.) You are offered the following wager: a single card is drawn from a standard deck of 52 cards. If the card is an Ace, you win $10; otherwise you lose $1.

a.) Make a table showing the possible values for x and their associated probabilities.

x	−1	10
Probability	$\frac{48}{52}$	$\frac{4}{52}$

b.) If you played this game many, many times, would be your expected financial outcome? (Express your answer in dollars and cents.)

$-1\left(\frac{48}{52}\right)+10\left(\frac{4}{52}\right)=-\frac{8}{52}=-.1538=$ $\boxed{\text{player will lose 15 cents per game}}$

6.) The table below shows the probability distribution for samples of five dentists and the use of nitrous oxide (laughing gas) in their practice.

x = number of dentists using laughing gas	0	1	2	3	4	5
Probability	0.0102	0.0768	0.2304	0.3456	.2592	0.0778

a.) What is the probability of 4 dentists using laughing gas?

$.0102+.0768+.2304+.3456+.0778=.7408 \rightarrow 1-.7408=\boxed{.2592}$

b.) If you randomly selected 5 dentists, how many would we expect to use laughing gas?

$0(.0102)+1(.0768)+2(.2304)+3(.3456)+4(.2592)+5(.0778)=\boxed{3.0002 \text{ dentists}}$

c.) What is the probability that no more than 3 of the 5 dentists use laughing gas?

$.0102+.0768+.2304+.3456=\boxed{.663}$

7.) Let the output of the random variable x denote the number of defective computer parts in a shipment of 400. The following table gives the probability distribution of function of x.

x = number of defective computer parts	0	1	2	3	4	5
Probability	0.02	0.20	.30	.30	0.10	0.08

a.) What is the probability of a computer with 2 and 3 defective parts if there probability is the same?

$.02+.20+.10+.08=.4 \rightarrow 1-.4=.6\div 2=\boxed{.30 \text{ each}}$

b.) How many shipments have at least 3 defective parts?

$.3+.10+.08=.48\times 400=\boxed{192 \text{ shipments}}$

c.) In a typical shipment, how many defective computer parts would you find?

$0(.02)+1(.20)+2(.30)+3(.30)+4(.10)+5(.08)=\boxed{2.5 \text{ computer parts}}$