

Probability – Binomial Experiments (Expansions)

- **binomial experiments (expansions)** → used to find probabilities where there are 2 possible outcomes where key words to look for to know when to use this technique are Exactly, Almost, At least

Ex: What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice (A – D possible answer choices) quiz if you guess at every question? →

$$P(4 \text{ right}, 1 \text{ wrong}) = {}^5C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^1 = \boxed{1.5\%}$$

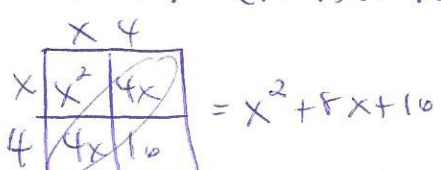
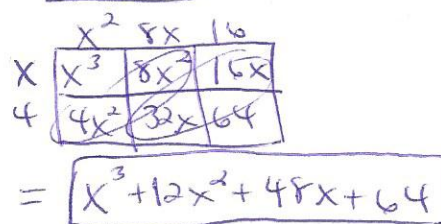
Binomial Exp "Formula" → total trials $C_{\text{successes}}$ · (successes prob)^{success power} · (failure prob)^{failures power}

Example 1: Find each probability using the Binomial Experiment "Formula".

<p>a.) If a family has 4 children, what is the probability that they have exactly 3 boys?</p> $P(\text{exactly 3 boys}) \rightarrow$ $= {}^4C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^1$ $= \boxed{25\%}$	<p>b.) Suppose that a coin is tossed 5 times, what is the probability of getting exactly 2 heads?</p> $P(\text{exactly 2 heads})$ $= {}^5C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3$ $= \boxed{31.3\%}$	
<p>c.) A die is rolled 3 times, what is the probability of getting exactly three 5's?</p> $P(\text{exactly three 5's}) \rightarrow$ $= {}^3C_3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^0$ $= \boxed{.46\%}$	<p>d.) Tarin and Sam are playing a certain board game, the probability of Tarin will a game is 75%. If they play 5 games, then what is the probability that Sam will win exactly 3 games?</p> $P(\text{Sam win exactly 3 games}) \rightarrow$ $= {}^5C_3 \cdot (.25)^3 \cdot (.75)^2$ $= \boxed{8.8\%}$	
<p>e.) Suppose that when hockey star Jamarie Jones takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game one night.</p>		
<p>i.) What is the probability that he will score exactly 1 goal?</p> $P(\text{exactly 1 goal})$ $= {}^6C_1 \cdot \left(\frac{1}{7}\right)^1 \cdot \left(\frac{6}{7}\right)^5$ $= \boxed{39.7\%}$	<p>ii.) What is the probability that he will score at most 2 goals?</p> $P(0) + P(1) + P(2) \rightarrow$ $= {}^6C_0 \cdot \left(\frac{1}{7}\right)^0 \cdot \left(\frac{6}{7}\right)^6$ $+ {}^6C_1 \cdot \left(\frac{1}{7}\right)^1 \cdot \left(\frac{6}{7}\right)^5$ $+ {}^6C_2 \cdot \left(\frac{1}{7}\right)^2 \cdot \left(\frac{6}{7}\right)^4$ $= \boxed{95.8\%}$	<p>iii.) What is the probability that he will score at least 4 goals?</p> $P(4) + P(5) + P(6) \rightarrow$ $= {}^6C_4 \cdot \left(\frac{1}{7}\right)^4 \cdot \left(\frac{6}{7}\right)^2$ $+ {}^6C_5 \cdot \left(\frac{1}{7}\right)^5 \cdot \left(\frac{6}{7}\right)^1$ $+ {}^6C_6 \cdot \left(\frac{1}{7}\right)^6 \cdot \left(\frac{6}{7}\right)^0$ $= \boxed{.49\%}$

– **Pascal's Triangle** → a special way and technique to expand a binomial expression BUT it can also be used to perform binomial experiments

Let's look at a simple example: $(x+4)^3 \rightarrow$ Produce the answer through 3 different techniques below.

Box Method	Pascal's Triangle	Binomial Experiments
$(x+4)^2 = (x+4)(x+4)(x+4)$  $= x^2 + 8x + 16$  $= x^3 + 12x^2 + 48x + 64$	$ \begin{array}{cccc} & & 1 & \rightarrow (x+4)^0 \\ & 1 & & 1 \rightarrow (x+4)^1 \\ & 1 & 2 & 1 \rightarrow (x+4)^2 \\ 1 & 3 & 3 & 1 \rightarrow (x+4)^3 \end{array} $ $ \begin{aligned} &= 1(x)^3(4)^0 + 3(x)^2(4)^1 \\ &+ 3(x)^1(4)^2 + 1(x)^0(4)^3 \\ &= \boxed{x^3 + 12x^2 + 48x + 64} \end{aligned} $	$ \begin{aligned} &{}_3C_0 (x)^3 (4)^0 = 1 \cdot x^3 \cdot 1 \\ &{}_3C_1 (x)^2 (4)^1 = 3 \cdot x^2 \cdot 4 \\ &{}_3C_2 (x)^1 (4)^2 = 3 \cdot x \cdot 16 \\ &{}_3C_3 (x)^0 (4)^3 = 1 \cdot 1 \cdot 64 \\ &= \boxed{x^3 + 12x^2 + 48x + 64} \end{aligned} $

Example 2: Complete each problem below using binomial experiments.

<p>a.) Expand: $(2x+5)^4$</p> $ \begin{aligned} &{}_4C_0 (2x)^4 (5)^0 = 1 \cdot 16x^4 \cdot 1 \\ &{}_4C_1 (2x)^3 (5)^1 = 4 \cdot 8x^3 \cdot 5 \\ &{}_4C_2 (2x)^2 (5)^2 = 6 \cdot 4x^2 \cdot 25 \\ &{}_4C_3 (2x)^1 (5)^3 = 4 \cdot 2x \cdot 125 \\ &{}_4C_4 (2x)^0 (5)^4 = 1 \cdot 1 \cdot 625 \end{aligned} $ $= \boxed{16x^4 + 160x^3 + 600x^2 + 1000x + 625}$	<p>b.) Find 3rd term: $(x^3-2)^7$</p> $ \begin{aligned} &{}_7C_0 (x^3)^7 (-2)^0 \\ &{}_7C_1 (x^3)^6 (-2)^1 \\ &{}_7C_2 (x^3)^5 (-2)^2 \end{aligned} $ <p style="text-align: center;">↓</p> $= 21 \cdot x^{15} \cdot 4$ $= \boxed{84x^{15}}$	<p>c.) Find middle term: $(3x^2-4)^{10}$</p> <p style="text-align: center;">$\frac{10}{2} = 5$</p> $ \begin{aligned} &{}_{10}C_5 (3x^2)^5 (-4)^5 \\ &= 252 \cdot 243x^{10} \cdot (-1024) \\ &= \boxed{-62705664x^{10}} \end{aligned} $
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