Probability – Binomial Experiments (Expansions)

- **binomial experiments (expansions)** → used to find probabilities where there are 2 possible outcomes where key words to look for to know when to use this technique are Exactly, Atmost, At least

Ex: What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice (A – D possible answer choices) quiz if you guess at every question? →

$P(4 \text{ right}, 1 \text{ wrong}) = {}_5C_4 \cdot (\frac{1}{4})^4 \cdot (\frac{3}{4})^1 = \boxed{1.5\%}$

Binomial Exp "Formula" → ${}_{\text{total trials}}C_{\text{successes}} \cdot (\text{successes prob})^{\text{success power}} \cdot (\text{failure prob})^{\text{failures power}}$

Example 1: Find each probability using the Binomial Experiment "Formula".

a.) If a family has 4 children, what is the probability that they have exactly 3 boys?

$P(\text{exactly } 3 \text{ boys}) \rightarrow$

$= {}_4C_3 \cdot (\frac{1}{2})^3 (\frac{1}{2})^1$

$= \boxed{25\%}$

b.) Suppose that a coin is tossed 5 times, what is the probability of getting exactly 2 heads?

$P(\text{exactly } 2 \text{ heads})$

$= {}_5C_2 (\frac{1}{2})^2 \cdot (\frac{1}{2})^3$

$= \boxed{31.3\%}$

c.) A die is rolled 3 times, what is the probability of getting exactly three 5's?

$P(\text{exactly three } 5\text{'s}) \rightarrow$

$= {}_3C_3 (\frac{1}{6})^3 \cdot (\frac{5}{6})^0$

$= \boxed{.46\%}$

d.) Tarin and Sam are playing a certain board game, the probability of Tarin will a game is 75%. If they play 5 games, then what is the probability that Sam will win exactly 3 games?

$P(\text{Sam win exactly } 3 \text{ games}) \rightarrow$

$= {}_5C_3 \cdot (.25)^3 (.75)^2$

$= \boxed{8.8\%}$

e.) Suppose that when hockey star Jamarie Jones takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game one night.

i.) What is the probability that he will score exactly 1 goal?

$P(\text{exactly } 1 \text{ goal})$

$= {}_6C_1 \cdot (\frac{1}{7})^1 (\frac{6}{7})^5$

$= \boxed{39.7\%}$

ii.) What is the probability that he will score at most 2 goals?

$P(0) + P(1) + P(2) \rightarrow$

$= {}_6C_0 (\frac{1}{7})^0 (\frac{6}{7})^6$

$+ {}_6C_1 (\frac{1}{7})^1 (\frac{6}{7})^5$

$+ {}_6C_2 (\frac{1}{7})^2 (\frac{6}{7})^4$

$= \boxed{95.8\%}$

iii.) What is the probability that he will score at least 4 goals?

$P(4) + P(5) + P(6) \rightarrow$

$= {}_6C_4 (\frac{1}{7})^4 (\frac{6}{7})^2$

$+ {}_6C_5 (\frac{1}{7})^5 (\frac{6}{7})^1$

$+ {}_6C_6 (\frac{1}{7})^6 (\frac{6}{7})^0$

$= \boxed{.49\%}$

– **Pascal's Triangle** → a special way and technique to expand a binomial expression BUT it can also be used to perform binomial experiments

Let's look at a simple example: $(x + 4)^3$ → Produce the answer through 3 different techniques below.

Box Method	Pascal's Triangle	Binomial Experiments
$(x+4)^3 = (x+4)(x+4)(x+4)$ $\begin{array}{c\|c\|c} & x & 4 \\ \hline x & x^2 & 4x \\ \hline 4 & 4x & 16 \end{array} = x^2+8x+16$ $\begin{array}{c\|c\|c\|c} & x^2 & 8x & 16 \\ \hline x & x^3 & 8x^2 & 16x \\ \hline 4 & 4x^2 & 32x & 64 \end{array}$ $= \boxed{x^3+12x^2+48x+64}$	$1 \rightarrow (x+4)^0$ $1 \quad 1 \rightarrow (x+4)^1$ $1 \quad 2 \quad 1 \rightarrow (x+4)^2$ $1 \quad 3 \quad 3 \quad 1 \rightarrow (x+4)^3$ $= 1(x)^3(4)^0 + 3(x)^2(4)^1$ $+ 3(x)^1(4)^2 + 1(x)^0(4)^3$ $= \boxed{x^3+12x^2+48x+64}$	${}_3C_0(x)^3(4)^0 = 1 \cdot x^3 \cdot 1$ ${}_3C_1(x)^2(4)^1 = 3 \cdot x^2 \cdot 4$ ${}_3C_2(x)^1(4)^2 = 3 \cdot x \cdot 16$ ${}_3C_3(x)^0(4)^3 = 1 \cdot 1 \cdot 64$ $= \boxed{x^3+12x^2+48x+64}$

Example 2: Complete each problem below using binomial experiments.

a.) Expand: $(2x + 5)^4$ ${}_4C_0(2x)^4(5)^0 = 1 \cdot 16x^4 \cdot 1$ ${}_4C_1(2x)^3(5)^1 = 4 \cdot 8x^3 \cdot 5$ ${}_4C_2(2x)^2(5)^2 = 6 \cdot 4x^2 \cdot 25$ ${}_4C_3(2x)^1(5)^3 = 4 \cdot 2x \cdot 125$ ${}_4C_4(2x)^0(5)^4 = 1 \cdot 1 \cdot 625$ $= \boxed{16x^4 + 160x^3 + 600x^2 + 1000x + 625}$	b.) Find 3rd term: $(x^3 - 2)^7$ ${}_7C_0(x^3)^7(-2)^0$ ${}_7C_1(x^3)^6(-2)^1$ ${}_7C_2(x^3)^5(-2)^2$ ↓ $= 21 \cdot x^{15} \cdot 4$ $= \boxed{84x^{15}}$	c.) Find middle term: $(3x^2 - 4)^{10}$ $\frac{10}{2} = 5$ ${}_{10}C_5(3x^2)^5(-4)^5$ $= 252 \cdot 243x^{10} \cdot (-1024)$ $= \boxed{-62705664x^{10}}$