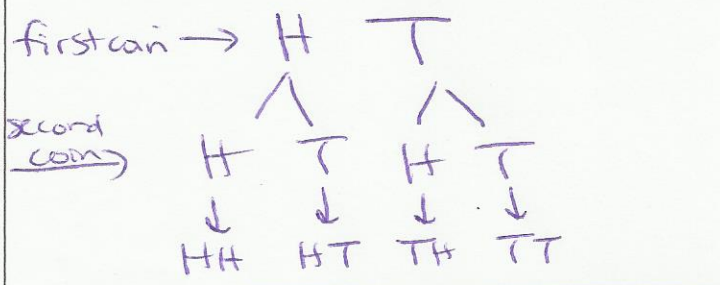
Probability – Basic Probability Concepts

- **probability** → represents the fraction of $\frac{\text{desired outcome}}{\text{total outcome}}$

- The probability of an event occurring is always between 0 and 1 inclusive
  - The closer the probability of an event is closer to 1 then the more likely the event will occur
  - The closer the probability of an event is closer to 0 then the less likely the event will occur
- Probability can be represented using a tree diagram or using combinations (nCr) (when selecting more than one)
- Probability is stated as a number in three forms: fraction, decimal, and percent (see most often)

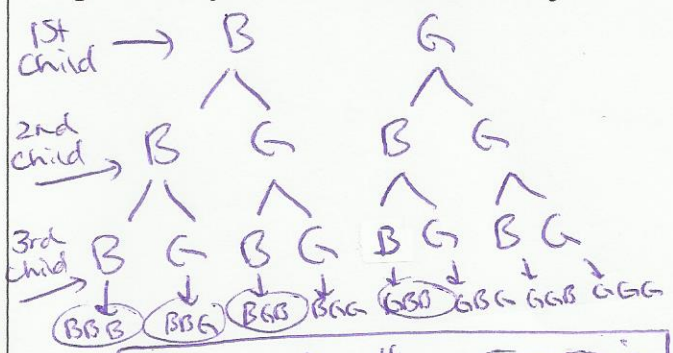
Example 1: Determine the probability using a tree diagram in its three forms.

a.) When two coins are tossed, what is the probability that there will be at least a tail?



$P(\text{at least a tail}) = \frac{3}{4} = .75 = 75\%$

b.) A woman wants three children, what is the probability that she will have 2 boys?



$P(\text{2 boys}) = \frac{3}{8} = .5 = 50\%$

Example 2: Determine the probability using combinations as a percent.

a.) Ebony has 4 male kittens and 7 female kittens. She picks up 2 kittens to give to a friend. Find the probability for the following: (nCr; 11 kittens total)

i.) P (2 male)

$\frac{4C_2}{11C_2} = \frac{6}{55} = 10.9\%$

ii.) P (2 female)

$\frac{7C_2}{11C_2} = \frac{21}{55} = 38.2\%$

iii.) P (1 of each)

$\frac{4C_1 \cdot 7C_1}{11C_2} = \frac{28}{55} = 50.9\%$

iv.) P (1 male, 2 female)

3 kittens

0%

She only picked up 2 kittens

b.) Bob is moving and all of his CDs are mixed up in a box. Twelve CDs are rock, eight are jazz, and five are classical. If he reaches into the box and selects them at random, find the probability for... (25 CDs total)

i.) P (1 classical)

$\frac{5}{25} = 20\%$

ii.) P (3 jazz)

$\frac{8C_3}{25C_3} = \frac{56}{2300} = 2.4\%$

iii.) P (2 classical, 1 rock)

$\frac{5C_2 \cdot 12C_1}{25C_3} = \frac{120}{2300} = 5.2\%$

iv.) P (2 jazz, 3 reggae)

no reggae CDs

0%

v.) P (1 classical, 1 jazz, 2 rock)

$\frac{5C_1 \cdot 8C_1 \cdot 12C_2}{25C_4} = \frac{2640}{12650} = 20.9\%$

Example 3: **Use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) in April 2000.**

If a student taking the test were randomly selected, find each probability.
Please express answer as a rounded percent.

a.) P (math or statistics) $\frac{179}{25405} = .7\%$

b.) P (biological sciences) $\frac{15819}{25405} = 62.3\%$

c.) P (social sciences or humanities) $\frac{2482+963}{25405} = \frac{3445}{25405} = 13.6\%$

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

Total = 25405 students

– **geometric probability** → represents the fraction of $\frac{\text{AREA of desired outcome}}{\text{AREA of total outcome}}$

Common Area Formulas You SHOULD KNOW:

Area of Square → $\text{Area} = (\text{side})^2$

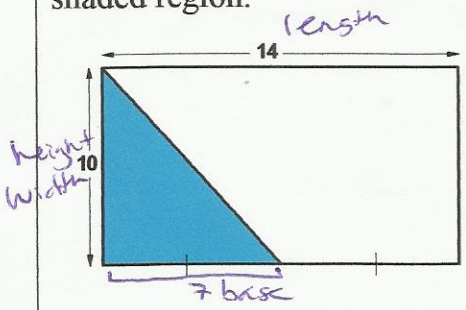
Area of Rectangle → $\text{Area} = \text{length} \times \text{width}$

Area of Triangle → $\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$

Area of Circle → $\text{Area} = (\text{radius})^2 \pi$

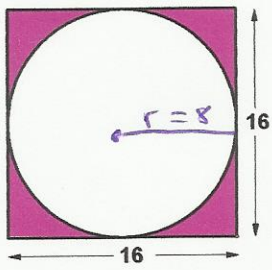
Example 4: **Find the geometric probability for each given situation/diagram. Express as a percent.**

a.) A coin is thrown from a ladder.
Find the probability of the coin landing in the shaded region.



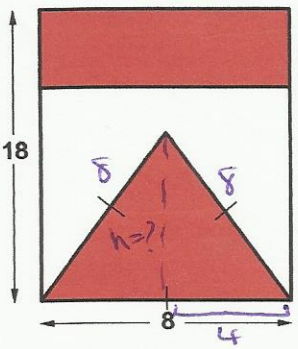
$\frac{\text{area of } \triangle}{\text{area of } \square} = \frac{\frac{1}{2}(7)(10)}{14 \cdot 10} = \frac{35}{140} = 25\%$

b.) A rock is thrown from a second story building.
Find the probability of the rock landing in the shaded region.



$\frac{\text{area of } \square - \text{area of } \bigcirc}{\text{area of } \square} = \frac{16^2 - 8^2\pi}{16^2} = \frac{54.938}{256} = 21.5\%$

c.) A dart is thrown at the dart board below. Find the probability that the dart landed in the shaded region.



$\frac{\text{area } \square + \text{area of } \triangle}{\text{area of } \square} = \frac{(8)(4) + \frac{1}{2}(8)(6.9)}{(8)(18)} = \frac{32 + 27.6}{144} = \frac{59.6}{144} = 41.4\%$

$h^2 + 4^2 = 8^2$, $h = 6.9$

d.) An archery target has 5 scoring zones formed by concentric circles. The radius of the yellow zone is 12.2 cm and the width of each ring is also 12.2 cm. If an arrow hits the target at a random point, what is the probability that it hits any area of the red zone?

→ r = 61 cm for big ○

$\frac{\text{area of red} - \text{area of yellow}}{\text{area of big } \bigcirc} = \frac{(24.4^2\pi) - (12.2^2\pi)}{(61^2\pi)} = \frac{1402.78}{11689.87} = 12\%$