Advanced Functions – Arithmetic Sequences and Series Word Problems Examples

Use the following formulas for the word problems below: $a_n = a_1 + (n-1)\cdot d$ and $S_n = \frac{n}{2}(a_1 + a_n)$

1.) According to the National Education Association, teachers in the US earned an average of $21,700 per year in 1984. This amount increased by approximately $1,472 yearly. Determine the amount a US teacher earned in the year 2005.

$a_1 = 21700 \quad n = 2005 - 1984 = 21$
$d = 1472 \quad a_{21} = ?$

$a_{21} = 21700 + (21-1)\cdot 1472$

$a_{21} = \$51,140$

2.) A drive-in theater has spaces for 20 cars in the first parking row, 22 in the second, 24 in the third, and so on. If there are 34 rows in the theater, find the number of cars that can be parked.

$d = 2$
$a_1 = 20, a_2 = 22, a_3 = 24$
$n = 34, a_{34} = ? \rightarrow S_{34} = ?$

$a_{34} = 20 + (34-1)\cdot 2$
$a_{34} = 86$

$S_{34} = \frac{34}{2}(20 + 86)$

$S_{34} = 1802$ seats

3.) The purchase value of an office computer is $12,500. The value of the computer after 6 years is $1,850. What was the computer's annual depreciation?

$a_1 = 12500 \quad n = 6+1$ (factor in 1st year.)
$a_7 = 1850, \; d = ? \quad n = 7$

$1850 = 12500 + (7-1)\cdot d$
$1850 = 12500 + 6d$
$-12500 \quad -12500$
$-10650 = 6d$
$\frac{-10650}{6} = \frac{6d}{6}$

$d = -1775 \rightarrow \$1775$

4.) An architect designs a small theater with 15 seats in the first row, 18 in the second, 21 in the third, and 36 seats in the last row. If the theater is to have a seating capacity of 204, how many rows did the architect use in his design?

$a_1 = 15, \; a_2 = 18, \; a_3 = 21, \; a_n = 36$ ($3 = d$)
$S_n = 204, \; n = ?$

$204 = \frac{n}{2}(15 + 36)$
$204 = \frac{n}{2}(51)$
$\frac{204}{25.5} = \frac{25.5n}{25.5}$

$n = 8 \rightarrow$ 8 rows

5.) A display of shoe boxes has 45 boxes the fourth row and has 94 boxes in the eleventh row. How boxes are in the first row?

$a_4 = 45, \; a_{11} = 94$

$94 = a_1 + (11-1)\cdot d$
$45 = a_1 + (4-1)\cdot d$ } system of eq.

$-\begin{cases} 94 = a_1 + 10d \\ 45 = a_1 + 3d \end{cases}$

$\frac{49}{7} = \frac{7d}{7}$

$d = 7$

24 in 1st row

$a_4 = 45$
$a_3 = 45 - 7 = 38$
$a_2 = 38 - 7 = 31$
$a_1 = 31 - 7 = 24$

6.) A radio station considered giving away $4,000 every day in the month of August for a total of $124,000. Instead they decided to increase the amount given away every day while still giving away the same total amount. If they want to increase the amount by $100, how much should they give away the first day?

$n = 31, \; S_{31} = 124000, \; d = 100$

$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)\cdot d)$
$S_n = \frac{n}{2}(2a_1 + (n-1)\cdot d)$
$124000 = \frac{31}{2}(2a_1 + (31-1)\cdot 100)$
$2\cdot 124000 = \frac{31}{2}(2a_1 + 3000)\cdot 2$
$248000 = 31(2a_1 + 3000)$
$8000 = 2a_1 + 3000$
$\frac{5000}{2} = \frac{2a_1}{2}$

$a_1 = \$2500$