Sequences and Series – Arithmetic Series and Sigma Notation (By Hand)

Introduction to (General) Series

- **series** → the indicated sum of terms in a sequence

• distinguish the difference – sequence → 6 , 12 , 18 , … , 300 where it contains commas

series → 6 + 12 + 18 + … + 300 where it contains "+" signs (no commas)

• series notation – $\underset{\text{Sum of first n terms}}{S_n} = \underbrace{a_1 + a_2 + a_3 + \ldots + a_n}_{\text{Sum of 1st term thru nth term}}$

Example 1: Find the sum of the first four terms for $a_n = 3n + 5$.

1st term → $a_1 = 3(1)+5 = 8$
2nd term → $a_2 = 3(2)+5 = 11$
3rd term → $a_3 = 3(3)+5 = 14$
4th term → $a_4 = 3(4)+5 = 17$

$S_4 = 8 + 11 + 14 + 17$

$\boxed{S_4 = 50}$

Specific Series # 1 – Arithmetic Series

- **arithmetic series** → the indicated sum of terms in an arithmetic sequence

where it's represented by the following formula: $S_n = \frac{n}{2}(a_1 + a_n)$

S_n → sum of first n terms; a_1 → 1st term; a_n → nth term: $a_n = a_1 + (n-1)\cdot d$

Example 2: Find S_n for each arithmetic series described.

a.) $a_1 = 8$, $a_n = 85$, $n = 12$ $S_{12} = \frac{12}{2}(8+85)$ $S_{12} = 6(93)$ $\boxed{S_{12} = 558}$	b.) $a_1 = 21$, $d = -5$, $n = 13$ $a_{13} = ?$ $a_{13} = 21 + (13-1)\cdot -5$ $a_{13} = -39$ $S_{13} = \frac{13}{2}(21-39)$ $\boxed{S_{13} = -117}$	c.) $a_1 = -32$, $d = 4$, $a_n = 156$ $n = ?$ $156 = -32 + (n-1)\cdot 4$ $156 = -32 + 4n - 4$ $156 = 4n - 36$ $+36 \quad +36$ $\frac{192}{4} = \frac{4n}{4}$ $n = 48$ $S_{48} = \frac{48}{2}(-32+156)$ $\boxed{S_{48} = 2976}$
d.) $d = -12$, $n = 19$, $a_n = -231$ $a_1 = ?$ $-231 = a_1 + (19-1)\cdot -12$ $-231 = a_1 - 216$ $+216 \quad +216$ $a_1 = -15$ $S_{19} = \frac{19}{2}(-15-231)$ $\boxed{S_{19} = -2337}$	e.) 9 + 14 + 19 + … + 304 $n = ?$ a_1 $d = 5$ a_n $304 = 9 + (n-1)\cdot 5$ $304 = 9 + 5n - 5$ $304 = 5n + 4$ $-4 \quad -4$ $\frac{300}{5} = \frac{5n}{5}$ $n = 60$ $S_{60} = \frac{60}{2}(9+304)$ $\boxed{S_{60} = 9390}$	f.) The first 20 positive even integers $n = 20$ $2 + 4 + 6 + \ldots + ? = a_n$ a_1 $d = 2$ $a_{20} = 2 + (20-1)\cdot 2$ $a_{20} = 40$ $S_{20} = \frac{20}{2}(2+40)$ $\boxed{S_{20} = 420}$

Example 3: **Find the first three terms of an arithmetic series in which …**

$a_1 = 9$, $a_n = 105$, and $S_n = 741$. $\rightarrow n = ?$ + $d = ?$ then $a_2 = ?$, $a_3 = ?$

$741 = \frac{n}{2}(9+105)$

$741 = \frac{n}{2}(114)$

$\frac{741}{57} = \frac{57n}{57} \rightarrow n = 13$

$105 = 9 + (13-1) \cdot d$

$105 = 9 + 12d$

$-9 \quad -9$

$\frac{96}{12} = \frac{12d}{12}$

$d = 8$

$a_1 = 9$

$a_2 = 9 + 8 = 17$

$a_3 = 17 + 8 = 25$

$\boxed{9, 17, 25}$

Sigma Notation (By Hand Method)

- **sigma notation** → a less lengthy and more concise way to write out a series

• The following is a simple representation of Sigma Notation:

$$\sum_{n=1}^{4} 3n = 3(1) + 3(2) + 3(3) + 3(4)$$

$$3 + 6 + 9 + 12 \Rightarrow S_4 = 30$$

→ the Greek symbol $\sum$ means " the sum of "

→ "n" is the main variable (in this problem) and is called the index of summation

underneath Sigma → "1" is the first value of n and is known as the lower limit

above Sigma → "4" is the last value of n and is known as the upper limit

right of Sigma → "3n" is the formula for the terms of the series

Example 4: **Find the sum of the given arithmetic series using the two methods as described.**
Both methods should be the SAME EXACT ANSWER.
Make sure to write your answer as S_n = sum where you fill in "n" and the series' sum.

Given arithmetic series → $\sum_{j=5}^{10} 4j - 7$

Method # 1 – Sigma Notation (by hand)	Method # 2 – Arithmetic Series Formula
$= [4(5)-7] + [4(6)-7] + [4(7)-7] + [4(8)-7] + [4(9)-7] + [4(10)-7]$ $= \underbrace{13 + 17 + 21 + 25 + 29 + 33}_{\text{6 terms to add up so } n = 6}$ (also find n by upper − lower + 1 → 10 − 5 + 1 = 6) $\Rightarrow \boxed{S_6 = 138}$	$S_n = \frac{n}{2}(a_1 + a_n)$ (n=5) $a_1 = 4(5) - 7 = 13$ (n=10) $a_6 = 4(10) - 7 = 33$ $S_6 = \frac{6}{2}(13 + 33)$ $\boxed{S_6 = 138}$

matches ✓