

Adv Functions - Arithm Series / Sigma Not (by hand) w/

1) $a_n = 2n - 6, S_3 = ?$

$$a_1 = 2(1) - 6 = -4$$

$$a_2 = 2(2) - 6 = -2$$

$$a_3 = 2(3) - 6 = 0$$

$$S_3 = -4 - 2 + 0$$

$$\boxed{S_3 = -6}$$

2) $a_n = 4 - 5n, S_5 = ?$

$$a_1 = 4 - 5(1) = -1$$

$$a_2 = 4 - 5(2) = -6$$

$$a_3 = 4 - 5(3) = -11$$

$$a_4 = 4 - 5(4) = -16$$

$$a_5 = 4 - 5(5) = -21$$

$$S_5 = -1 - 6 - 11 - 16 - 21$$

$$\boxed{S_5 = -55}$$

3) $a_n = \frac{1}{2}n + 3, S_4 = ?$

$$a_1 = \frac{1}{2}(1) + 3 = \frac{7}{2}$$

$$a_2 = \frac{1}{2}(2) + 3 = 4$$

$$a_3 = \frac{1}{2}(3) + 3 = \frac{9}{2}$$

$$a_4 = \frac{1}{2}(4) + 3 = 5$$

$$S_4 = \frac{7}{2} + 4 + \frac{9}{2} + 5$$

$$\boxed{S_4 = 17}$$

4) $a_n = 7 + \frac{1}{4}n, S_6 = ?$

$$a_1 = 7 + \frac{1}{4}(1) = \frac{29}{4}$$

$$a_2 = 7 + \frac{1}{4}(2) = \frac{15}{2}$$

$$a_3 = 7 + \frac{1}{4}(3) = \frac{31}{4}$$

$$a_4 = 7 + \frac{1}{4}(4) = 8$$

$$a_5 = 7 + \frac{1}{4}(5) = \frac{33}{4}$$

$$a_6 = 7 + \frac{1}{4}(6) = \frac{17}{2}$$

$$S_6 = \frac{29}{4} + \frac{15}{2} + \frac{31}{4} + 8 + \frac{33}{4} + \frac{17}{2}$$

$$\boxed{S_6 = \frac{189}{4} \text{ or } 47.25}$$

5) $a_1 = 16, a_n = 98, n = 13 \rightarrow S_n = ?$

$$S_{13} = \frac{13}{2}(16 + 98)$$

$$S_{13} = \frac{13}{2}(114)$$

$$\boxed{S_{13} = 741}$$

6) $a_1 = 5, n = 11, d = 4 \rightarrow a_n = ?$

$$a_{11} = 5 + (11-1) \cdot 4$$

$$S_n = ?$$

$$a_{11} = 45$$

$$S_{11} = \frac{11}{2}(5 + 45)$$

$$\boxed{S_{11} = 275}$$

$$7) a_n = 33, d = 2, a_1 = 5 \rightarrow n = ?$$

$$S_n = ?$$

$$33 = 5 + (n-1) \cdot 2$$

$$33 = 5 + 2n - 2$$

$$33 = 3 + 2n$$

$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$\begin{array}{r} 30 \\ 2 \end{array} = \begin{array}{r} 2n \\ 2 \end{array}$$

$$n = 15$$

$$S_{15} = \frac{15}{2} (5 + 33)$$

$$\boxed{S_{15} = 285}$$

$$8) d = -8, n = 16, a_n = -108 \rightarrow$$

$$a_1 = ?$$

$$S_n = ?$$

$$-108 = a_1 + (16-1) \cdot (-8)$$

$$-108 = a_1 + (15) \cdot (-8)$$

$$-108 = a_1 - 120$$

$$\begin{array}{r} +120 \\ +120 \end{array}$$

$$a_1 = 12$$

$$S_{16} = \frac{16}{2} (12 - 108)$$

$$\boxed{S_{16} = -768}$$

$$9) 8 + 15 + 22 + \dots + 155 \rightarrow n = ?$$

$$\begin{array}{l} \downarrow \downarrow \\ a_1 \quad d=7 \end{array}$$

$$\begin{array}{l} \downarrow \\ a_n \end{array} \quad S_n = ?$$

$$155 = 8 + (n-1) \cdot 7$$

$$155 = 8 + 7n - 7$$

$$155 = 1 + 7n$$

$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$\begin{array}{r} 154 \\ 7 \end{array} = \begin{array}{r} 7n \\ 7 \end{array}$$

$$n = 22$$

$$S_{22} = \frac{22}{2} (8 + 155)$$

$$\boxed{S_{22} = 1793}$$

$$10) 1 + 3 + 5 + \dots + a_{17} \rightarrow S_n$$

$$\begin{array}{l} \downarrow \downarrow \\ a_1 \quad d=2 \end{array}$$

$$a_{17} = 1 + (17-1) \cdot 2$$

$$a_{17} = 33$$

$$S_{17} = \frac{17}{2} (1 + 33)$$

$$\boxed{S_{17} = 289}$$

$$11) a_1 = 14, n = 21, d = -6 \rightarrow a_n = ?$$

$$S_n = ?$$

$$a_{21} = 14 + (21-1) \cdot (-6)$$

$$a_{21} = -106$$

$$S_{21} = \frac{21}{2} (14 - 106)$$

$$\boxed{S_{21} = -966}$$

$$12) d = \frac{2}{5}, n = 10, a_n = \frac{19}{5} \rightarrow a_1 = ?$$

$$S_n = ?$$

$$\frac{19}{5} = a_1 + (10-1) \cdot \frac{2}{5}$$

$$\frac{19}{5} = a_1 + (9) \cdot \frac{2}{5}$$

$$\frac{19}{5} = a_1 + \frac{18}{5}$$

$$\begin{array}{r} -18 \\ -18 \end{array} \quad \begin{array}{r} 19 \\ 19 \end{array}$$

$$a_1 = \frac{1}{5}$$

$$S_{10} = \frac{10}{2} \left(\frac{1}{5} + \frac{19}{5} \right)$$

$$\boxed{S_{10} = 20}$$

$$13) 3 + 6 + 9 + \dots + 78 \rightarrow n = ?$$

$$\begin{array}{l} \downarrow \downarrow \\ a_1 \quad d=3 \end{array}$$

$$\begin{array}{l} \downarrow \\ a_n \end{array}$$

$$S_n = ?$$

$$78 = 3 + (n-1) \cdot 3$$

$$78 = 3 + 3n - 3$$

$$S_{26} = \frac{26}{2} (3 + 78)$$

$$\begin{array}{r} 78 \\ 3 \end{array} = \begin{array}{r} 3n \\ 3 \end{array}$$

$$n = 26$$

$$\boxed{S_{26} = 1053}$$

$$14) a_n = 148, a_1 = -20, n = 25 \rightarrow S_n = ?$$

$$S_{25} = \frac{25}{2} (-20 + 148)$$

$$\boxed{S_{25} = 1600}$$

$$16) 89 + 86 + 83 + \dots + 20 \rightarrow n = ?$$

$$\downarrow \quad \downarrow$$

$$a_1 = -3 \quad a_n$$

$$20 = 89 + (n-1) \cdot (-3)$$

$$20 = 89 - 3n + 3$$

$$20 = 92 - 3n$$

$$\underline{-92} \quad \underline{-92}$$

$$\underline{-72} = \underline{-3n}$$

$$\underline{-3} \quad \underline{-3}$$

$$n = 24$$

$$S_{24} = \frac{24}{2} (89 + 20)$$

$$\boxed{S_{24} = 1308}$$

$$18) n = 16, a_n = 15, S_n = -120$$

$$\rightarrow a_1 = ?, d = ?, a_2 = ?, a_3 = ?$$

$$-120 = \frac{16}{2} (a_1 + 15)$$

$$-120 = 8(a_1 + 15)$$

$$-120 = 8a_1 + 120$$

$$\underline{-120} \quad \underline{-120}$$

$$\underline{-240} = \underline{8a_1}$$

$$\underline{8} \quad \underline{8}$$

$$a_1 = -30$$

$$a_1 = -30$$

$$a_2 = -30 + 3 = -27$$

$$a_3 = -27 + 3 = -24$$

$$\boxed{-30, -27, -24}$$

$$15 = -30 + (16-1) \cdot d$$

$$15 = -30 + 15d$$

$$\underline{+30} \quad \underline{+30}$$

$$\underline{45} = \underline{15d}$$

$$\underline{15} \quad \underline{15}$$

$$d = 3$$

$$15) a_1 = -121, d = 3, a_n = 5$$

$$\rightarrow n = ? \quad S_n = ?$$

$$S = -121 + (n-1) \cdot 3$$

$$S = -121 + 3n - 3$$

$$S = -124 + 3n$$

$$\underline{+124} \quad \underline{+124}$$

$$\underline{129} = \underline{3n}$$

$$\underline{3} \quad \underline{3}$$

$$n = 43$$

$$S_{43} = \frac{43}{2} (-121 + 5)$$

$$\boxed{S_{43} = -2494}$$

$$17) a_1 = 14, a_n = -85, S_n = -1207$$

$$\rightarrow n = ?, d = ?, a_2 = ?, a_3 = ?$$

$$-1207 = \frac{n}{2} (14 - 85)$$

$$2 \cdot -1207 = \frac{n}{2} (-71) \cdot 2$$

$$\underline{-2414} = \underline{-71n}$$

$$\underline{-71} \quad \underline{-71}$$

$$n = 34$$

$$a_1 = 14$$

$$a_2 = 14 - 3 = 11$$

$$a_3 = 11 - 3 = 8$$

$$\boxed{14, 11, 8}$$

$$-85 = 14 + (34-1) \cdot d$$

$$-85 = 14 + 33d$$

$$\underline{-14} \quad \underline{-14}$$

$$\underline{-99} = \underline{33d}$$

$$\underline{33} \quad \underline{33}$$

$$d = -3$$

19) $\sum_{k=1}^5 5+3k \rightarrow$	#1 - By Hand	#2 - Formula
	$[5+3(1)] + [5+3(2)] +$ $[5+3(3)] + [5+3(4)] + [5+3(5)]$ $= 8 + 11 + 14 + 17 + 20$ $\Rightarrow \boxed{S_5 = 70} \quad \checkmark$	$a_1 = 5 + 3(1) = 8$ $a_5 = 5 + 3(5) = 20$ $S_5 = \frac{5}{2}(8+20)$ $\boxed{S_5 = 70} \quad \checkmark$

20) $\sum_{n=4}^{11} 2n-3 \rightarrow$	#1 - By Hand	#2 - Formula
	$[2(4)-3] + [2(5)-3] + [2(6)-3]$ $+ [2(7)-3] + [2(8)-3] + [2(9)-3]$ $+ [2(10)-3] + [2(11)-3]$ $= 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ $\Rightarrow \boxed{S_8 = 96} \quad \checkmark$	$a_1 = 2(4) - 3 = 5$ $a_8 = 2(11) - 3 = 19$ $S_8 = \frac{8}{2}(5+19)$ $\boxed{S_8 = 96} \quad \checkmark$

21) $\sum_{j=3}^8 5j+2 \rightarrow$	#1 - By Hand	#2 - Formula
	$[5(3)+2] + [5(4)+2]$ $+ [5(5)+2] + [5(6)+2]$ $+ [5(7)+2] + [5(8)+2]$ $= 17 + 22 + 27 + 32 + 37 + 42$ $\Rightarrow \boxed{S_6 = 177} \quad \checkmark$	$a_1 = 5(3) + 2 = 17$ $a_6 = 5(8) + 2 = 42$ $S_6 = \frac{6}{2}(17+42)$ $\boxed{S_6 = 177} \quad \checkmark$

22) $18 + 22 + 26 + \dots + 62 = 492$?
 $\downarrow \quad \downarrow$
 $a_1 \quad d=4$
 \downarrow
 $a_n \quad n=? \quad S_n=?$

$$62 = 18 + (n-1) \cdot 4$$

$$62 = 18 + 4n - 4$$

$$62 = 14 + 4n$$

$$\underline{-14} \quad \underline{-14}$$

$$\frac{48}{4} = \frac{4n}{4} \quad n=12$$

$$S_{12} = \frac{12}{2}(18+62)$$

$$S_{12} = 480$$

\Rightarrow

No, the sum of the given series equals 480 and not 492.

23.) a) $5 + 7 + 9 + 11$ $\left\{ \begin{array}{l} 5(2) + 7(2) + 9(2) + 11(2) \\ = 10 + 14 + 18 + 22 \\ \Rightarrow S_4 = 64 \text{ which equals } 32(2) = 64 \end{array} \right.$

True

b) $5 + 7 + 9 + 11$ $\left\{ \begin{array}{l} 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 \\ \Rightarrow S_8 = 96 \text{ which equals } 32(3) = 96 \\ \downarrow \\ \text{not doubled!} \end{array} \right.$

False

24) $36^\circ + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 180^\circ$ where $n=3$ since there are 3 angles in a triangle.

A.) $180 = \frac{n}{2} (36 + 60)$ B.) $180 = \frac{n}{2} (36 + 72)$

$180 = \frac{n}{2} (96)$

$180 = \frac{n}{2} (108)$

$\frac{180}{48} = \frac{48n}{48}$

$\frac{180}{54} = \frac{54n}{54}$

$n = 3.75 \times$

$n = 3.33 \times$

C.) $180 = \frac{n}{2} (36 + 84)$
 $180 = \frac{n}{2} (120)$

D.) $180 = \frac{n}{2} (36 + 144)$

$180 = \frac{n}{2} (180)$

$\frac{180}{60} = \frac{60n}{60}$

$\frac{180}{90} = \frac{90n}{90}$

$n = 3 \checkmark$

$n = 2 \times$

$84 = 36 + (3-1) \cdot d$

$84 = 36 + 2d$

$a_1 = 36$

$\frac{48}{2} = \frac{2d}{2}$

$a_2 = 36 + 24 = 60$

$d = 24$

$a_3 = 60 + 24 = 84$

$S_3 = 36 + 60 + 84 = 180 \checkmark$

25.) $6 + 12 + 18 + \dots + 54$
 $\downarrow \quad \downarrow$
 $a_1 \quad d=6 \quad a_n$

$6(1) + 6(2) + 6(3) + \dots + 6(9)$

$\sum_{n=1}^9 6n$