

Sequences and Series – Arithmetic Sequences

Introduction to (General) Sequences


- **sequence** → a list of a pattern in a certain order
where each part of the pattern is called a term

• sequence notation – $a_1, a_2, a_3, \dots, a_n$

a_1 → first term
 a_2 → second term
 a_3 → third term
 a_n → n th term – function of a sequence at n

* term # " n " represents a positive whole number *

- sequence representations – we can represent a sequence in three ways:

1.) picture → Ex: 

2.) Words → Ex: marching, adjusting, thanking, hammering, matching, boarding
(texting), (holding)

3.) numbers → Ex: 3, 10, 24, 45, 73, 108
 $+7$ $+14$ $+21$ $+28$ $+35$

Specific Sequence # 1 – Arithmetic Sequence

Before Getting Started – The following are examples of arithmetic sequences:

1.) 12, 26, 40, 54, 68, 82 Pattern Rule: adding 14

2.) $\frac{5}{3}, \frac{7}{6}, \frac{2}{3}, \frac{1}{6}, \underline{-\frac{1}{3}}, \underline{-\frac{5}{6}}$ Pattern Rule: Subtracting $\frac{1}{2}$ (or adding $-\frac{1}{2}$)

What do you think happens in arithmetic sequences? keep adding/subtracting same #

- **arithmetic sequence** → a sequence where the difference between any two consecutive terms is a constant, called d, the common difference

(REMEMBER: *consecutive numbers* are #s next to each other)

Example 1: Determine if these sequences are arithmetic. If so, state the common difference.

a.) $3, 7, 11, 15, \dots$

b.) $4, -1, -6, -11, \dots$

c.) $-2, 4, 10, 13, \dots$

$$\begin{array}{r} \checkmark \quad \checkmark \quad \checkmark \\ 7-3 \quad 11-7 \quad 15-11 \\ =4 \quad =4 \quad =4 \end{array}$$
$$\begin{array}{r} \checkmark \\ -1-4 \\ =-5 \end{array} \quad \begin{array}{r} \checkmark \\ -6-1 \\ =-5 \end{array} \quad \begin{array}{r} \checkmark \\ -11-6 \\ =-5 \end{array}$$
$$\begin{array}{ccc} \checkmark & \checkmark & \checkmark \\ 4-2 & 16-4 & 13-10 \\ =6 & =6 & =3 \end{array}$$

Yes $\rightarrow d=4$

Yes $\rightarrow d = -5$

$$ND \rightarrow d = N/A$$

Example 2: Find the next four terms of each arithmetic sequence. Write your answer as sequence.

a.) $a_1 = 6$ and $d = 17$

b.) 55, 49, 43, ...

→ 55, 49, 43, 37, 31, 25, 19, ...

$$a_1 = 6$$
$$a_5 = 6 + 17 = 23$$
$$a_2 = 23 + 17 = 40$$
$$a_4 = 40 + 17 = 57$$
$$a_5 = 57 + 17 = 74$$

49-55 43-49

$$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

So $d = -6$

$$a_4 = 43 - 6 = 37$$
$$a_c = 37 - 6 = 31$$
$$a_{10} = 31 - 6 = 25$$
$$A - B = 25 - 6 = 19$$

"Nth Term Formula" of Arithmetic Sequence: Used to find ANY term of an arithmetic sequence

Consider an arithmetic sequence whose first term is a_1 and whose common difference is d :

a_1	→	1st term (a_1)
$a_1 + d$	→	2nd term ($a_2 = a_1 + d$)
$a_1 + d + d = a_1 + 2d$	→	3rd term ($a_3 = a_1 + 2d$)
$a_1 + 2d + d = a_1 + 3d$	→	4th term ($a_4 = a_1 + 3d$)
$a_1 + 3d + d = a_1 + 4d$	→	5th term ($a_5 = a_1 + 4d$)

(General) n^{th} term Formula: $a_n = a_1 + (n-1) \cdot d$ → some important notes about this formula...

- formula will always be a linear equation
- make sure your final n^{th} term formula is completely simplified
(distribute + combine like terms)

Example 3: Find what is indicated for each arithmetic sequence.

a.) $a_1 = -6$ and $d = 7$, find the 16 th term $n=16$ $a_{16} = -6 + (16-1) \cdot 7$ $a_{16} = -6 + (15) \cdot 7$ $a_{16} = 99$	b.) Find a_{40} for the sequence $-9, -17, -25, \dots$ $n=40$ $a_1 - 17 - 9 = -8 = d$ $a_{40} = -9 + (40-1) \cdot -8$ $a_{40} = -9 + (39) \cdot -8$ $a_{40} = -321$	c.) Write the n^{th} term formula (equation) for the sequence $8, 17, 26, 35, \dots$ $a_1 - 17 - 8 = 9 = d$ $a_n = 8 + (n-1) \cdot 9$ $a_n = 8 + 9n - 9$ $a_n = 9n - 1$
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Example 4: Considering all given sequences are arithmetic – Find what is asked.

a.) The 20 th term of the sequence is 101 and the common difference is 3. What is the fourth term? $n=20, a_{20}=101, d=3$ $101 = a_1 + (20-1) \cdot 3$ $101 = a_1 + 57$ -57 $a_1 = 44$ $a_4 = 44 + (4-1) \cdot 3$ $a_4 = 53$	b.) What is the common difference for the sequence where the first term is 13 and the 28 th term is -149 ? $a_1 = 13, a_{28} = -149, d=?$ $-149 = 13 + (28-1) \cdot d$ $-149 = 13 + 27d$ -13 $-162 = 27d$ -6 $d = -6$	c.) Which term of the sequence $1, 5, 9, \dots$ is 97? $a_1 - 5 - 1 = 4 = d$ $97 = 1 + (n-1) \cdot 4$ $97 = 1 + 4n - 4$ $97 = 4n - 3$ $+3$ $100 = 4n$ 25 $n = 25$ 25 th term
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- **arithmetic means** → represent the terms between any two non successive terms of an arithmetic sequence

Ex: Circle the 3 arithmetic means between 30 and 74: 19, 30, 41, 52, 63, 74, 85, 96, ...

Example 5: Find the arithmetic means of the arithmetic sequence below.

16, 31, 46, 61, 76, 91

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

$16 + 15 \quad 31 + 15 \quad 46 + 15 \quad 61 + 15 \quad 76 + 15 = 91$

$91 = 16 + (6-1) \cdot d$
 $91 = 16 + 5d$
 -16
 $75 = 5d$
 15
 $d = 15$