Released Form

Student Name: Key

Spring 2013
North Carolina
Measures of Student Learning:
NC's Common Exams
Advanced Functions and Modeling

READY

Student Booklet

Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314

1 The table below shows the probability distribution of the number of televisions in each house in a community.

B

Televisions	Probability
0	0.04
1	0.38
2	0.27
3	x
4	y
5 or more	0.13

.69

? + .69 = 100%, or 1

What is the probability that a house in the community will have at least 3 televisions?

A 0.69

(B) 0.31

C 0.18

D 0.09

1 − .69 = .31

2 Anna and Zach each have $600 to invest. Anna's investments earn a rate of 10.5%, and Zach's investments earn a rate of 6.5%. ***Approximately***, how much more money will Anna have than Zach when Zach's investments are worth $900? (Assume continuous compounding.)

C

A $184

B $241

(C) $255

D $264

$A = Pe^{rt}$

Anna

$A = 600e^{.105(6.239)}$

$A = 1155$

$1155 - 900 = 255$

Zach

$900 = 600e^{.065t}$

$1.5 = e^{.065t}$

$\frac{\ln 1.5}{.065} = \frac{.065t}{.065}$

$t = 6.2379$

Go to the next page.

3 A solution's pH is given by the function $p(t) = -\log(t)$, where t is the hydronium ion concentration, in moles per liter. A sample of coffee has a pH of 5.0. What is the ***approximate*** hydronium ion concentration of the sample?

B

A 0.000001

B 0.00001

C 0.0001

D 0.001

$-\log t = 5$

$\log t = -5$

$t = 10^{-5}$

$t = .00001$

4 A sequence is shown below.

1, 0.1, 0.01, 0.001, 0.0001, . . .

What is the sum of the sequence?

B

A $1\frac{1}{10}$

B $1\frac{1}{9}$

C $1\frac{2}{9}$

D $1\frac{9}{10}$

infinite geometric

$S = \frac{a_1}{1-r}$

$S = \frac{1}{1-.1}$

$= \frac{10}{9} = 1\frac{1}{9}$

$a_1 = 1$

$r = \frac{.1}{1} = .1$

5 Which statement is true about the sequence shown below?

C not arithmetic! not geometric!

0, 4.5, 12, 22.5, . . .

no sense A The series converges because the limit of the sequence as n approaches infinity is infinity.

B The series converges because the limit of the sequence as n approaches infinity is 30.

C The series diverges because the limit of the sequence as n approaches infinity is infinity.

not sense D The series diverges because the limit of the sequence as n approaches infinity is 30.

6 A pharmaceutical company is creating a new cholesterol drug to prevent heart disease. The company must collect data by testing the drug before it is approved. Which would be the ***best*** method of data collection?

A

A experimental study

B observational study

C simulation

D survey

7 The table below shows the midterm and final exam grades of ten students.

A

Midterm	68	78	92	90	88	82	94	83	71	62
Final Exam	62	77	99	87	85	84	95	98	72	64

$\bar{x}=80.8\ \sigma_x=10.3$

$\bar{x}=82.3\ \sigma_x=12.6$

Which comparison between the midterm grades and the final exam grades is true?

(A) The final exam grades have a higher mean and standard deviation than the midterm grades.

B The final exam grades have a lower mean and standard deviation than the midterm grades.

C The final exam grades have a higher mean and a lower standard deviation than the midterm grades.

D The final exam grades have a lower mean and a higher standard deviation than the midterm grades.

8 A baseball team scored the following number of runs in its games this season: 6, 2, 5, 9, 11, 4, 5, 8, 6, 7, 5. There is one more game in the season. If the team wants to end the season with an average of at least 6 runs per game, what is the least number of runs the team must score in the final game of the season?

B

A 2

(B) 4

C 6

D 8

$\frac{68+x}{12} \geq 6$ (12)

$68+x=72$

$-68 \quad -68$

$x=4$

C

9 If the probability of giving birth to a boy is 0.52, what is the ***approximate*** probability of giving birth to four consecutive boys?

A 0.021

B 0.062

(C) 0.073

D 0.130

P(B, B, B, and B)

$= .52 \times .52 \times .52 \times .52$

$= .073$

10 How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative?

D

A 94,830

B 11,628

C 4,320

(D) 4,248

$10P_4$ | $12C_5$

5040 | 792

5040 − 792 = (4248)

11 A starting line for a hockey team should consist of 3 offensive players, 2 defensive players, and 1 goaltender. A coach has 11 offensive players, 6 defensive players, and 2 goaltenders from which to choose the starting line. How many unique starting lines can the coach create?

A 132

B 792

(C) 4,950

D 59,400

$11C_3 \times 6C_2 \times 2C_1$

$= 165 \times 15 \times 2$

$= (4950)$

12 It costs a bakery $3.50 to make apple pies that sell for $12 the first day they are baked.

- If a pie is not sold on the first day, the new price is $8.50.
- The probability of selling the apple pie the first day is 75%.
- There is a 12% probability of selling it on the second day.
- If the apple pie does not sell by the end of the second day, it is donated.

What is the ***approximate*** expected profit per pie for the bakery on the sale of its apple pies?

B

A $5.67

B $6.52

C $9.57

D $10.02

$12(.75) + 8.5(.12)$

$10.02 - 3.5$

$= 6.52$

13 The number of household members, x, living in Cityville homes has the following probability distribution:

B

x	1	2	3	4	5	6	7	8
$P(x)$	0.21	0.28	0.16	0.22	0.06	0.04	0.02	0.01

What is the expected size of a household in Cityville?

A 2.43

B 2.89

C 3.17

D 4.50

$1(.21) + 2(.28) + 3(.16) + 4(.22) + 5(.06) + 6(.04) + 7(.02) + 8(.01)$

$= 2.89$

14 What is the middle term for the expansion of $(x^2 + 3)^{12}$?

C

A $729x^{12}$

B $924x^{12}$

C $673{,}596x^{12}$

D $665{,}280x^{12}$

$12C_6(x^2)^6(3)^6$

$= 924(x^{12})(729)$

$= 673596x^{12}$

15 Abby took an 8-question multiple-choice quiz. Suppose that her probability of correctly answering any question is 0.75. What is Abby's probability of incorrectly answering exactly two questions on the quiz?

C

A P = 0.089

B P = 0.240

C P = 0.311

D P = 0.623

$8C_2(.25)^2(.75)^6$

$= .311$

16 Which function results by shifting the graph of $y = \ln(x + 3) - 6$ to the left 4 units and down 3 units?

A

A $y = \ln(x + 7) - 9$

B $y = \ln(x - 1) - 9$

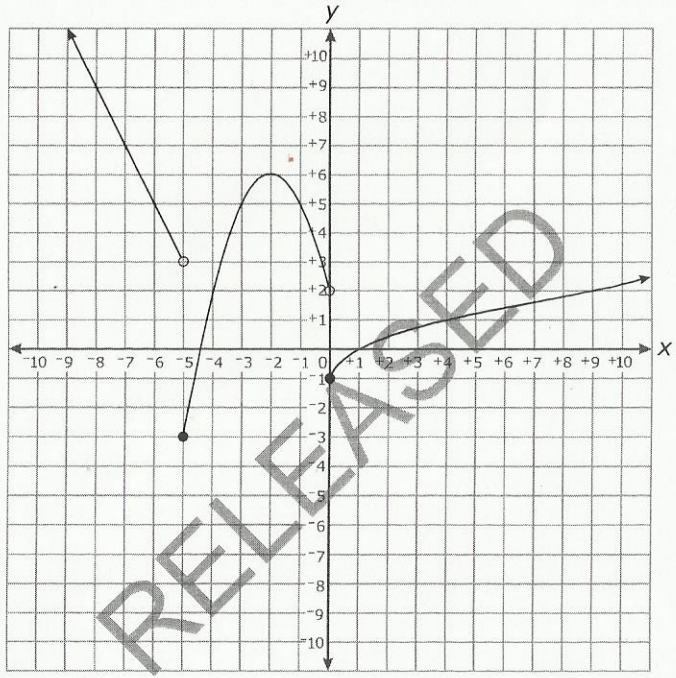
C $y = \ln(x + 7) - 3$

D $y = \ln(x - 1) - 3$

$+4 \quad -3$

$y = \ln(x+7) - 9$

17 Which piecewise function is graphed below?



Answer choices are on the following page.

A

A $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x+2)^2 + 6 & \text{for } -5 \le x < 0 \\ \sqrt{x} - 1 & \text{for } x \ge 0 \end{cases}$

B $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x-2)^2 + 6 & \text{for } -5 \le x < 0 \\ \sqrt{x} - 1 & \text{for } x \ge 0 \end{cases}$

C $f(x) = \begin{cases} -2x - 7 & \text{for } x \le -5 \\ -(x-2)^2 + 6 & \text{for } -5 < x \le 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$

D $f(x) = \begin{cases} -2x - 7 & \text{for } x \le -5 \\ -(x+2)^2 + 6 & \text{for } -5 < x \le 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$

18 A function, $f(x)$, is shown below.

$$f(x) = \begin{cases} x - 4 & \text{for } 0 \le x < 2 \\ x^2 - 3x + 4 & \text{for } 2 \le x < 4 \\ 5 & \text{for } 4 \le x < 7 \end{cases}$$

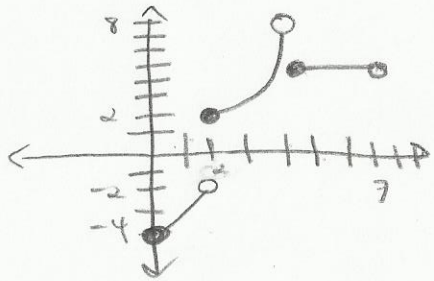
What is the range of $f(x)$?

A $[-4, 5)$

B $[-4, 8)$

C $[-4, -2) \cup [2, 5)$

D $[-4, -2) \cup [2, 8)$



19 A water tower is located 410 feet from a building. From a window in the building, it is observed that the angle of elevation to the top of the tower is 42 degrees and the angle of depression to the bottom of the tower is 25 degrees. ***Approximately*** how tall is the water tower?

D

A 191 feet

B 369 feet

C 448 feet

(D) 560 feet

$h = 369 + 191 = 560$

$\tan 42 = \frac{x}{410}$

$x = 369$

$\tan 25 = \frac{y}{410}$

$y = 191$

20 Given the table below:

x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
y	0.5	0	-0.5	0	0.5

Which function fits the data?

(A) $y = 0.5\cos(2x - \pi)$

~~B~~ $y = 0.5\cos(x - \pi)$

C $y = 0.5\cos\left(2x + \frac{\pi}{2}\right)$

~~D~~ $y = \cos\left(2x + \frac{\pi}{2}\right)$

amp = .5

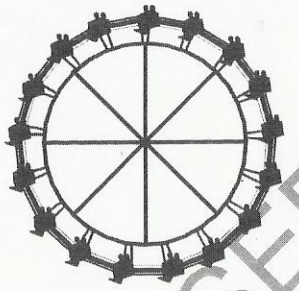
$\text{period} = \frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2} = \pi$

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi}$

$b = 2$

21 A Ferris wheel has a diameter of 80 feet. Riders enter the Ferris wheel at its lowest point, 5 feet above the ground, at time $t = 0$ seconds. One complete rotation takes 65 seconds.

B



Which function models a rider's vertical height, $h(t)$, at t seconds?

A $h(t) = -80\cos\left(\frac{2\pi}{65}t\right) + 5$

(B) $h(t) = -40\cos\left(\frac{2\pi}{65}t\right) + 45$

C $h(t) = -45\cos\left(\frac{65}{2\pi}t\right) + 40$

D $h(t) = -5\cos\left(\frac{65}{2\pi}t\right) + 80$

$a = \frac{80}{2} = 40$

$b = \frac{2\pi}{65}$

$d = 40 + 5 = 45$

22 How does the graph of $g(x) = 0.5\cos(2x)$ differ from the graph of its parent function, $f(x) = \cos(x)$, over the interval $-\pi \le x \le \pi$?

A (A) The amplitude is smaller, and the period is shorter.

B The amplitude is smaller, and the period is longer.

C The amplitude is larger, and the period is shorter.

D The amplitude is larger, and the period is longer.

$y = .5\cos(2x)$

$amp = \frac{1}{2}$

$period = \frac{2\pi}{2} = \pi$

$y = \cos x$

$amp = 1$ $period = 2\pi$

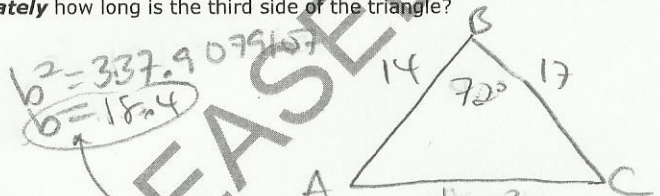
23 Two sides of a triangle measure 14 ft and 17 ft, respectively. The included angle is 72°. ***Approximately*** how long is the third side of the triangle?

A (A) 18.4 ft

B 20.3 ft

C 25.1 ft

D 30.7 ft



$b^2 = 337.9079107$

$b = 18.4$

$b^2 = 14^2 + 17^2 - 2(14)(17)\cos 72°$

24 In a geometric sequence, $a_1 = 12$ and $r = \sqrt{2}$. What is the ***approximate*** sum of the first 20 terms of the sequence?

D A 339.4

B 8,688.9

C 29,624.9

(D) 29,636.9

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$= \frac{12(1-\sqrt{2}^{20})}{(1-\sqrt{2})}$

≈ 29636.9

25 A bathroom floor has tiles arranged in 9 circles. The innermost circle contains 9 tiles. Each successive circle contains 9 more tiles than the previous circle. How many total tiles are on the bathroom floor?

C A 81

B 396

(C) 405

D 729

$d = 9$ (arithmetic)

$S = \frac{n}{2}(a_1 + a_n)$

This is the end of the multiple-choice portion of the test.

$a_9 = 9 + (9-1)\cdot 9$

$a_9 = 81$

$S_9 = \frac{9}{2}(9 + 81)$

$= 405$

The questions you read next will require you to answer in writing.

1. **Write your answers on separate paper.**
2. **Be sure to write your name on each page.**

L1 | L2

1 The table below shows the estimated average hours each person in a city spent playing video games in different years.

Years since 2002	Hours
0	71
1	80
2	82
3	78
4	80
5	91
6	107
7	121
8	125
9	131
10	142

- a) Write an equation for the best fit exponential model for the data.
- b) What is the meaning of the base of the model in the context of the problem?
- c) What is the meaning of the coefficient of the model in the context of the problem?

a) $y = 68.4858(1.0745)^x$

b) city and # hrs playing games is growing since base b > 1

c) represent appr initial amt of people playing games in city

2 Students are told that attending class regularly will help improve their scores in that class. Below are the scores for students who did attend class regularly and scores for those who did not.

Attended Class Regularly					Did Not Attend Class Regularly			
241	261	271	282	296	185	195	195	228
243	262	272	284	296	250	256	225	261
254	267	278	292	308	274	277	308	233
252	264	276	290	310				

185 195 195 225 228 233 250 256 261 274 277 308

med = 274 $\bar{x}$ = 274.95 med = 241.5 $\bar{x}$ = 240.58

- a) Which group of students has a larger mean score and by how much?
- b) Which group of students has a larger median score and by how much?

a) Attend class reg has larger mean by 34.37

b) Attend class reg has larger median by 32.5

3 Write an equation for the power function, in $y = ax^b$ form, that passes through the points (2, 1) and (5, 6).

- Use your power function to predict the value of y when $x = 9$.

x1 y1 x2 y2

x	y
2	1
5	6

} regression wkst tool

$y = ax^b$

$\frac{1}{2^b} = \frac{a(2)^b}{2^b}$

$a = \frac{1}{2^b}$

$a = \frac{1}{1.955}$

$a = .258$

$6 = a(5)^b$

$6 = \left(\frac{1}{2^b}\right).5^b$

$6 = \left(\frac{5}{2}\right)^b$

$\frac{\log 6}{\log\left(\frac{5}{2}\right)} = \frac{b\log\left(\frac{5}{2}\right)}{\log\left(\frac{5}{2}\right)}$

$b = 1.955$

$y = .258(1.955)^b \rightarrow y = .258(1.955)^9$

$y = 107.6$