Advanced Functions – 8.5 WS: Trig Equations and its Apps Name : Key

I. Solve each trigonometric equation. Keep answer(s) in degrees where $0° \le x < 360°$. Show work!!

1.) $2\cos x - \sqrt{3} = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = 30°, 330°$

2.) $2\sin x + 1 = 4\sin x$

$-2\sin x \quad -2\sin x$

$1 = 2\sin x$

$\frac{1}{2} = \frac{2\sin x}{2}$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = 30°, 150°$

3.) $\tan x + \sqrt{3} = 0$

$-\sqrt{3} \quad -\sqrt{3}$

$\tan x = -\sqrt{3}$

$x = \tan^{-1}(-\sqrt{3})$

$x = -60 + 360$

$x = 300°$

$120°$

4.) $(\tan x - 1)(\cos x + 2) = 0$

$\tan x - 1 = 0 \qquad \cos x + 2 = 0$

$\tan x = 1 \qquad \cos x = -2$

$x = \tan^{-1}(1) \qquad x = \cos^{-1}(-2)$

$x = 45°, 225°$ $\qquad$ $\emptyset$

5.) $4\sin\left(\frac{x}{5}\right) - 4 = 0$

$\frac{4\sin\left(\frac{x}{5}\right)}{4} = \frac{4}{4}$

$\sin\left(\frac{x}{5}\right) = 1$

$\frac{x}{5} = \sin^{-1}(1)$

$\frac{x}{5} = 90°$

$x = 450°$

$\emptyset$

no solution

6.) $3\tan 3x - \sqrt{3} = 0$

$+\sqrt{3} \quad +\sqrt{3}$

$\frac{3\tan 3x}{3} = \frac{\sqrt{3}}{3}$

$\tan 3x = \frac{\sqrt{3}}{3}$

$3x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\frac{3x}{3} = \frac{30° + 180°}{3}$

$x = 10°, 70°$

7.) $2\cos\left(\frac{3x}{5}\right) + \sqrt{2} = 0$

$\frac{2\cos\left(\frac{3x}{5}\right)}{2} = \frac{-\sqrt{2}}{2}$

$\cos\left(\frac{3x}{5}\right) = -\frac{\sqrt{2}}{2}$

$\frac{3x}{5} = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\frac{3x}{5} = 135°, 225°$

$\frac{3x}{3} = \frac{675}{3}, \frac{1125}{3}$

$x = 225°$ $\qquad$ $3375°$ $\emptyset$

8.) $\sin x(1 - \sin x) = 0$

$\sin x = 0 \qquad 1 - \sin x = 0$

$x = \sin^{-1}(0) \qquad -\sin x = -1$

$x = 0°, 180°$ $\qquad$ $(\sin x = 1)\sin^{-1}$

$x = \sin^{-1}(1)$

$x = 90°$

9.) $3\cos x + 8 = 3 - 2\cos x$

$+2\cos x \quad -3 \quad -3 \quad +2\cos x$

$\frac{5\cos x}{5} = \frac{-5}{5}$

$\cos x = -1$

$x = \cos^{-1}(-1)$

$x = 180°$

10.) $2\sin(2x + 30°) = 1$

$\sin(2x + 30°) = \frac{1}{2}$

$2x + 30 = \sin^{-1}\left(\frac{1}{2}\right)$

$2x + 30 = 30°, 150°$

$\frac{2x}{2} = \frac{0° + 120°}{2}$

$x = 0°, 60°$

11.) $(2\sin x + 1)(\sqrt{3} + 3\tan x) = 0$

$2\sin x + 1 = 0 \qquad \sqrt{3} + 3\tan x = 0$

$2\sin x = -1 \qquad \frac{3\tan x}{3} = \frac{-\sqrt{3}}{3}$

$\sin x = -\frac{1}{2} \qquad \tan x = -\frac{\sqrt{3}}{3}$

$x = \sin^{-1}\left(-\frac{1}{2}\right) \qquad x = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$x = -30 + 360$

$x = 330°, 210°$ $\qquad$ $x = 330°, 150°$

12.) $\frac{1}{\cos x} - \frac{2}{\sqrt{3}}$

$-2\cos x = \sqrt{3}$

$\frac{-2\cos x}{-2} = \frac{\sqrt{3}}{-2}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$x = 150°, 210°$

13.) $\cos x(\cos x + 1)(\sqrt{3}\tan x - 1) = 0$

$\cos x = 0 \qquad \cos x + 1 = 0$

$x = \cos^{-1}(0) \qquad (\cos x = -1)\cos^{-1}$

$x = 90°, 270°$ $\qquad$ $x = \cos^{-1}(-1)$

$x = 180°$

$\sqrt{3}\tan x - 1 = 0$

$\sqrt{3}\tan x = 1$

$\left(\tan x = \frac{1}{\sqrt{3}}\right)\tan^{-1}$

$x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$x = 30°, 210°$

14.) $\frac{1}{2\sin x - 3} \times \frac{2}{3\sin x}$

$2(2\sin x - 3) = 3\sin x$

$4\sin x - 6 = 3\sin x$

$-3\sin x \quad +6 \quad -3\sin x \quad +6$

$\sin x = 6$

$x = \sin^{-1}(6)$

$\emptyset$ → no solution

15.) $4\sin^2 x - 3 = 0$

$\frac{4\sin^2 x}{4} = \frac{3}{4}$

$\sqrt{\sin^2 x} = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \qquad x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$x = 60°, 120°$ $\qquad$ $x = 300°, 240°$

II. Complete each application problem involving a trigonometric equation. Show work!! (work on separate sheet)

16.) A Mercator project map uses a flat projection of Earth in which the distance between the lines of latitude increases with their distance from the equator. The calculation of the location of a point on this projection P uses the formula $P = \tan\left(\theta + \frac{L}{2}\right)$, where L is the latitude of the point. What is the measure of angle θ if the latitude of a point is 60° and the location of the point is 3.73?

$\theta = 45°$

17.) Rhonda wants to wait to plant her flowers until there at least 12 hours of daylight. The number of daylight H in her town can be represented by $H = 11.45 + 6.5\sin(0.0168d - 1.333)$, where d is the day of the year and angle measures are in radians. On what day is it safe for Rhonda to plant her flowers?

$d = 84$ days

→ March 25

18.) A ball on a spring is pulled 4 inches below its rest position and then released. After t seconds, the ball's distance from its rest position is given by $d = -4\cos(60°t)$. What are all the values of t for which the ball is 3 inches above its rest position?

$t = 0.3 + 1n$

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16) $P = \tan(\theta + \frac{5}{2})$

$3.73 = \tan(\theta + \frac{60}{2})$

$\tan^{-1}(3.73 = \tan(\theta + 30^\circ))\tan^{-1}$

$75^\circ = \theta + 30^\circ$

$-30^\circ \quad -30^\circ$

$\boxed{\theta = 45^\circ}$

17) MODE = Radians!! — for this problem only.

$H = 11.45 + 6.5\sin(0.0168d - 1.333)$

$12 = 11.45 + 6.5\sin(0.0168d - 1.333)$

$-11.45 \quad -11.45$

$\frac{.55}{6.5} = \frac{6.5\sin(.0168d - 1.333)}{6.5}$

$\sin^{-1}(.0846153 = \sin(.0168d - 1.333))$

$.0847166823 = .0168d - 1.333$

$+1.333 \quad +1.333$

$\frac{1.4}{.0168} = \frac{.0168d}{.0168}$

$d = 84$ days → March 25th

18) $d = -4\cos(60^\circ t)$

$\frac{3}{-4} = \frac{-4\cos(60^\circ t)}{-4}$

$\cos^{-1}(-.75 = \cos(60^\circ t))\cos^{-1}$

$\frac{138.6^\circ}{60^\circ} = \frac{60^\circ t}{60^\circ}$

$60^\circ = \frac{\pi}{3}$

$\text{period} = \frac{2\pi}{\frac{\pi}{3}}$

$b = 6$

$\boxed{t = 2.3 + 6n}$