

8.4 – Writing Equations for Sine and Cosine Functions

Sine Function → $y = a \sin(bx \pm c) \pm d$

Cosine Function → $y = a \cos(bx \pm c) \pm d$

where use the following “formulas” to find each parameter (the value of each letter):

$a = \text{amplitude}$ $b = \frac{2\pi}{\text{period}}$ $c = -b \cdot \text{ph. shift}$ $d = \begin{matrix} \rightarrow \text{up} = +d \\ \rightarrow \text{down} = -d \end{matrix}$

Example 1: Given specific set of details about a function, write its equation.

Given Info	Function	Work to Find a, b, c, and d	Equation for Function
a.) amplitude = 4, period = $\frac{\pi}{3}$, left by $\frac{\pi}{2}$, up by 5	sine	$a = 4$ $b = \frac{2\pi}{\frac{\pi}{3}} = 6$ $c = -6 \cdot -\frac{\pi}{2} = 3\pi$ $d = 5$	$y = 5 \sin(6x + 3\pi) + 5$
b.) amplitude = 1, period = 4π , right by 2π , down by 3	cosine	$a = 1$ $b = \frac{2\pi}{4\pi} = \frac{1}{2}$ $c = -\frac{1}{2} \cdot 2\pi = -\pi$ $d = -3$	$y = \cos(\frac{1}{2}x - \pi) - 3$

Example 2: Given a specific graph of a function, write its equation. → **midline**

Given Graph	Info About Graph	Work to Find a, b, c, and d	Equation for Function
a.) 	3 dots line up → $y = \sin x$ amplitude = 3 period = π p. shift = none v. shift = down 1	$a = 3$ $b = \frac{2\pi}{\pi} = 2$ $c = -2 \cdot 0 = 0$ $d = -1$	$y = 3 \sin(2x) - 1$
b.) 	2 dots line up → $y = \cos x$ amplitude = 2 period = 2π p. shift = left $\frac{\pi}{2}$ v. shift = none	$a = 2$ $b = \frac{2\pi}{2\pi} = 1$ $c = -1 \cdot -\frac{\pi}{2} = \frac{\pi}{2}$ $d = 0$	$y = 2 \cos(x + \frac{\pi}{2})$
c.) 	2 dots line up → $y = \cos x$ amplitude = 4 period = 8π p. shift = right 2π v. shift = up 2	$a = 4$ $b = \frac{2\pi}{8\pi} = \frac{1}{4}$ $c = -\frac{1}{4} \cdot 2\pi = -\frac{\pi}{2}$ $d = 2$	$y = 4 \cos(\frac{1}{4}x - \frac{\pi}{2}) + 2$
d.) 	3 dots line up → $y = \sin x$ amplitude = 3 period = $\frac{\pi}{2}$ p. shift = left $\frac{\pi}{4}$ v. shift = down 2	$a = 3$ $b = \frac{2\pi}{\frac{\pi}{2}} = 4$ $c = -4 \cdot -\frac{\pi}{4} = \pi$ $d = -2$	$y = 3 \sin(4x + \pi) - 2$