

## 8.2 – The Unit Circle and Finding Exact Value (using the UC)

– **unit circle** → a circle with a radius of 1 and centered at (0, 0) and has equation of  $x^2 + y^2 = 1$

- **reference angle** → an acute angle formed between a drawn angle  $\theta$  and the x-axis.
- **terminal point** → a point (x, y) that falls on the Unit Circle.
- **cosine function** → represents the x-coordinate of the terminal point of an angle on the Unit Circle.
- **sine function** → represents the y-coordinate of the terminal point of an angle on the Unit Circle.

Refer to **TRIG CHART / UNIT CIRCLE SHEET** to label parts of the Unit Circle:

- 1.) Complete the TRIG CHART → Use the 45 – 45 – right  $\Delta$  and the 30 – 60 – right  $\Delta$   
For quadrant angles ( $0^\circ$  and  $90^\circ$ ), use your calculator
- 2.) Label the degree measure ABOVE each pt on the Unit Circle (only use increments of  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ )
- 3.) Label the radian measure BELOW each pt on the Unit Circle (convert degree measure to radians)
- 4.) Draw diagonal lines through pairs of points that have the same reference number (angle):
  - a.)  $30^\circ$  and  $210^\circ$  } **Red** Ref Angle =  $30^\circ$   
 $150^\circ$  and  $330^\circ$  }
  - b.)  $45^\circ$  and  $225^\circ$  } **Blue** Ref Angle =  $45^\circ$   
 $135^\circ$  and  $315^\circ$  }
  - c.)  $60^\circ$  and  $240^\circ$  } **Green** Ref Angle =  $60^\circ$   
 $120^\circ$  and  $300^\circ$  }
- 5.) Label the terminal point (x, y) of each degree/radian measure → ( $x = \cos \theta$ ,  $y = \sin \theta$ )
- 6.) Write in Quadrant #'s and where trig functions are positive (ALL SENIORS TAKE CALCULUS)

**Example 1: Using your TC/UC Sheet, answer each question.**

a.) What is the reference angle for the angle of $240^\circ$ ? $240^\circ - 180^\circ = 60^\circ$ (green)	b.) What is the reference angle for the angle of $\frac{3\pi}{4}$ ? $180 - 135 = 45^\circ = \frac{\pi}{4}$ (blue)	c.) What is the reference angle for the angle of $-750^\circ$ ? $360 - 330 = 30^\circ$ (red)
d.) What is the terminal point for the angle of $510^\circ$ ? $510^\circ - 360^\circ = 150^\circ$ $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$	e.) What is the terminal point for the angle of $-\frac{9\pi}{4}$ ? $-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$ $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$	f.) If you are at terminal pt $(-1, 0)$ and move $\frac{7\pi}{4}$ CW, what angle did you stop at that is on the UC? $180 - 315 = -135^\circ + 360^\circ = 225^\circ = \frac{5\pi}{4}$

**Steps to Find Exact Value of an Angle:** Some answers contain radicals (**NO decimal answers**)

- 1.) Find the reference angle B – Use the “Coloring Coding key” to help determine this.
  - 2.) Use Trig Chart to look up value using reference angle B.
  - 3.) Use “Signs” Diagram of Trigonometric Functions to determine is value is positive or negative
- \* If finding the exact value of a quadrant angle ( $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , or  $360^\circ$ ) → use values in terminal points

**Example 2: Using your TC/UC Sheet, find the exact value. Remember – NO DECIMALS!!!!**

a.) $\sin 135^\circ = \frac{\sqrt{2}}{2}$ II $45^\circ +$	b.) $\cos 210^\circ = -\frac{\sqrt{3}}{2}$ III $30^\circ -$	c.) $\cos 360^\circ = 1$ Quadrant (1) 0	d.) $\tan -780^\circ = -\sqrt{3}$ II $60^\circ -$
e.) $\sin 390^\circ = \frac{1}{2}$ I $30^\circ +$	f.) $\tan 270^\circ = \text{undefined}$ Quadrant (0, -1) $\rightarrow -1/0$	g.) $\tan 150^\circ = -\frac{\sqrt{3}}{3}$ II $30^\circ -$	h.) $\cos 315^\circ = \frac{\sqrt{2}}{2}$ IV $45^\circ +$
i.) $\tan\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ III $30^\circ +$	j.) $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ II $60^\circ -$	k.) $\sin(-3\pi) = 0$ -540 $\rightarrow$ 180 (-1, 0)	l.) $\tan\left(\frac{15\pi}{4}\right) = -1$ IV $45^\circ -$