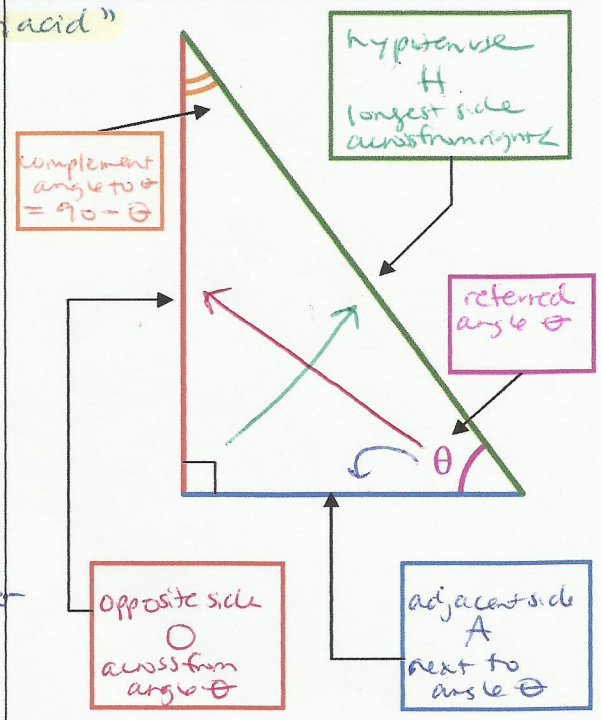
6.2 – Trigonometric Ratios in Right Triangles

The Three Basic Trigonometric Ratios in Right Triangles

"Some old hippie caught another hippie tripping on acid"

Word	Abbreviation	Definition
Sine	$\sin\theta$	$\sin\theta = \frac{O}{H}$
Cosine	$\cos\theta$	$\cos\theta = \frac{A}{H}$
tangent	$\tan\theta$	$\tan\theta = \frac{O}{A}$



Notes: 1.) The symbol θ (called Theta) is a Greek letter that is often used to refer or indicate angle(s).

2.) You may have to use Pyth. Thm to find third side.

– **rationalizing (the denominator)** → way to simplify an expression with radicals (square roots) by NOT ALLOWING radical(s) to be in denominator (we will have do this in Example 2) so let's review/practice this concept…

Example 1: Let's review on how to "Rationalize the Denominator".

a.) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ (= value of 1)

$= \frac{4\sqrt{5}}{\sqrt{25}} = \boxed{\frac{4\sqrt{5}}{5}}$

b.) $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{6\sqrt{3}}{\sqrt{9}} = \frac{{}^{2}6\sqrt{3}}{3_{1}} = \boxed{2\sqrt{3}}$

c.) $\frac{2}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

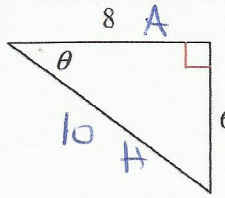
$= \frac{2\sqrt{2}}{4 \cdot 2} = \frac{2\sqrt{2}}{8_{4}} = \boxed{\frac{\sqrt{2}}{4}}$

d.) $\frac{48}{3\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{48\sqrt{6}}{3 \cdot 6} = \frac{{}^{8}\cancel{48}\sqrt{6}}{\cancel{18}_{3}} = \boxed{\frac{8\sqrt{6}}{3}}$

Example 2: Find the value of the three basic trigonometric ratios.

a.)



$6^2 + 8^2 = c^2$

$100 = c^2$

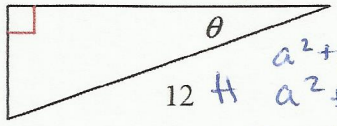
$c = 10$

$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{6}{8} = \frac{3}{4}$

b.)



O: 4

$a^2 + (8\sqrt{2})^2 = 12^2$

$a^2 + 128 = 144$

$a^2 = 16$

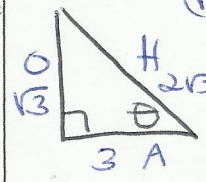
$a = 4$

$\sin\theta = \frac{4}{12} = \frac{1}{3}$

$\cos\theta = \frac{8\sqrt{2}}{12} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{4}{8\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{16} = \frac{\sqrt{2}}{4}$

c.) Given $\tan\theta = \frac{\sqrt{3}}{3}$ (O = $\sqrt{3}$, A = 3), find the other two trigonometric ratios.



$(\sqrt{3})^2 + (3)^2 = c^2$

$c^2 = 12$

$c = 2\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

$\cos\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

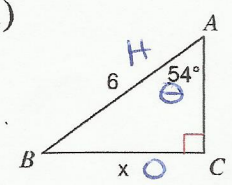
Check Your Calculator Mode - It needs to be in DEGREES!

Finding a Missing Side with a Trigonometric Ratio and Solving a Right Triangle

- With trigonometry, you only need to know one side and one angle (other than the right angle) in order to find the missing side x in a right triangle.
- Decide how the given side and given angle relates to the missing side x.
- Set up a(n) equation using the appropriate basic trig ratio.
  - If "x" is on top, then multiply → Ex: $\tan 56° = \frac{x}{21}$ $x = 21\tan 56$ → $x = 31.1$
  - If "x" is on bottom, then divide → Ex: $\sin 72° = \frac{16}{x}$ $x = \frac{16}{\sin 72}$ → $x = 16.8$
- Solving a right triangle means to find all missing side and angles → will have 3 answers
  - Capital letter and Theta always represent angles measurements
  - Lowercase letters always represent side lengths (which are across from its angle)
- Round all answers to NEAREST TENTH! NO RADICALS in these sets of problems!

Example 3: Find the value of side x. Round to tenth place.

a.)

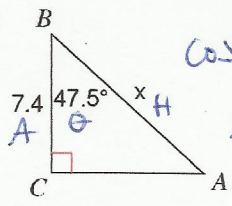


$\sin 54 = \frac{x}{6}$

$x = 6\sin 54$

$x = 4.9$

b.)

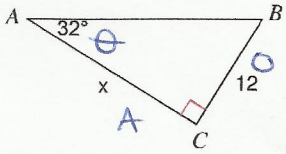


$\cos 47.5 = \frac{7.4}{x}$

$x = \frac{7.4}{\cos 47.5}$

$x = 11$

c.)



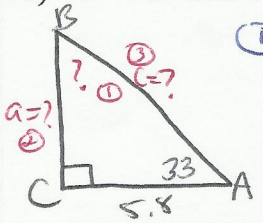
$\tan 32 = \frac{12}{x}$

$x = \frac{12}{\tan 32}$

$x = 19.2$

Example 4: Solve triangle ABC. Round to tenth place.

a.) A = 33° and b = 5.8



① $B = 90 - 33$

$B = 57°$

② $\tan 33 = \frac{a}{5.8}$

$a = 5.8\tan 33$

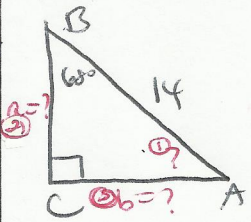
$a = 3.8$

③ $3.8^2 + 5.8^2 = c^2$

$c^2 = 47.08$

$c = 6.9$

b.) B = 68° and c = 14



① $A = 90 - 68$

$A = 22°$

② $\cos 68 = \frac{a}{14}$

$a = 14\cos 68$

$a = 5.2$

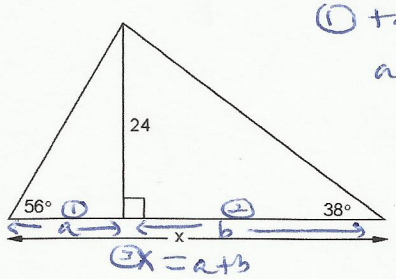
③ $5.2^2 + b^2 = 14^2$

$b^2 = 168.96$

$b = 13$

Example 5 – Critical Thinking: Find the length of side x. Round to tenth place.

a.)



① $\tan 56 = \frac{24}{a}$

$a = \frac{24}{\tan 56}$

$a = 16.2$

② $\tan 38 = \frac{24}{b}$

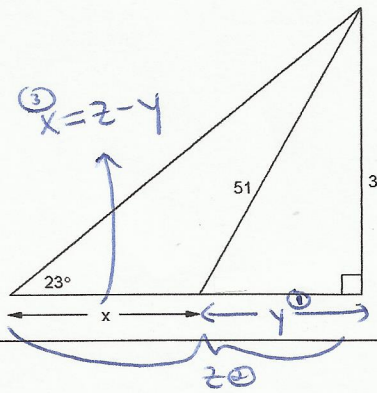
$b = \frac{24}{\tan 38}$

$b = 30.7$

③ $x = a + b$

$x = 16.2 + 30.7$ → $x = 46.9$

b.)



① $32^2 + y^2 = 51^2$

$y^2 = 1577$

$y = 39.7$

② $\tan 23 = \frac{32}{z}$

$z = \frac{32}{\tan 23}$ → $z = 75.4$

③ $x = z - y$

$x = 75.4 - 39.7$

$x = 35.7$