4.6 – Piecewise Functions

– **piecewise function** → a function and its graph is broken into different parts (hence its name) and different functions are ***defined on VARIOUS DOMAIN VALUES*** (x-values).

Step # 1 → Evaluate the function based on its domain (x) values ; start with #'s after the word IF.

Step # 2 → Make a table of values (points) ; indicate if those points are closed or open dots.

Step # 3 → After graphing (by hand – calculator is not helpful), CHECK for one MAJOR FEATURE: Piecewise function's graphs should NOT CROSS EACH OTHER!

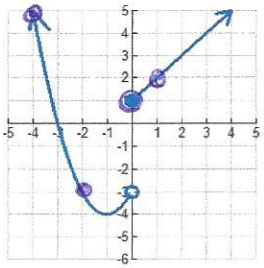
Example 1: Evaluate each piecewise function. Validate points with the function's given graph.

Given Piecewise Function

a.)

$$f(x)=\begin{cases} x^2+2x-3 & \text{if } x<0 \\ x+1 & \text{if } x\geq 0 \end{cases}$$

Graph of Piecewise Function

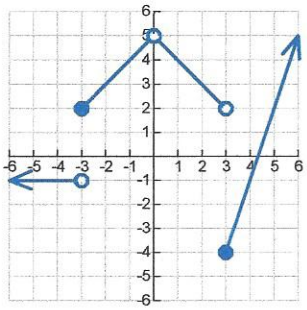


Evaluate/Complete Table of Values

x	Work to find f (x) or y	Pt (x,y)
−4	$(-4)^2+2(-4)-3=5$	$(-4,5)$
−2	$(-2)^2+2(-2)-3=-3$	$(-2,-3)$
0	$0+1=1$	$(0,1)$
1	$1+1=2$	$(1,2)$

b.)

$$f(x)=\begin{cases} -1 & \text{if } x<-3 \\ -|x|+5 & \text{if } -3\leq x<3 \cup x\neq 0 \\ 3x-13 & \text{if } x\geq 3 \end{cases}$$

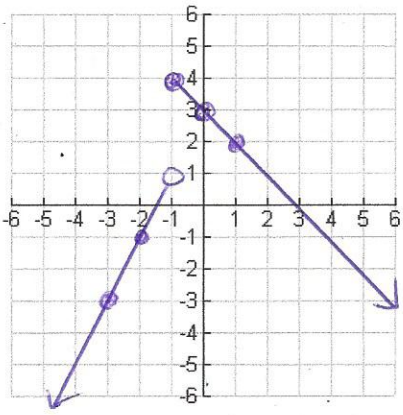


x	Work to find f (x) or y	Pt (x,y)
4	$3(4)-13=-1$	$(4,-1)$
−3	$-\|-3\|+5=2$	$(-3,2)$
−5	-1	$(5,-1)$
3	$3(3)-13=-4$	$(3,-4)$
0	undefined	$\emptyset$

Example 2: Make a table of domain values ("work") and graph each piecewise function.

a.)

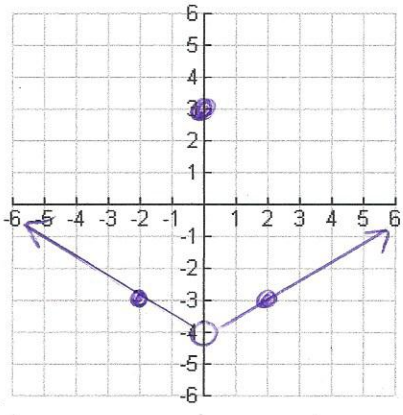
$$f(x)=\begin{cases} 2x+3 & \text{if } x<-1 \\ 3-x & \text{if } x\geq -1 \end{cases}$$



$2x+3$	$3-x$
$(-1,1)$ ○	$(-1,4)$ ●
$(-2,-1)$ ●	$(0,3)$ ●
$(-3,-3)$ ●	$(1,2)$ ●

b.)

$$f(x)=\begin{cases} \frac{1}{2}|x|-4 & \text{if } x\neq 0 \\ 3 & \text{if } x=0 \end{cases}$$



$\frac{1}{2}\|x\|-4$	3
$(0,-4)$ ○	$(0,3)$ ●
$(-2,-3)$ ●	
$(2,-3)$ ●	

Example 2 Cont'd: Make a table of domain values ("work") and graph each piecewise function.

c.)
$$f(x)=\begin{cases}3 & \text{if } x\le -2\\ 2x-1 & \text{if } -2<x\le 1\\ 4-x & \text{if } x>1\end{cases}$$

$(x\le -2)\ (-2<x\le 1)\ (x>1)$

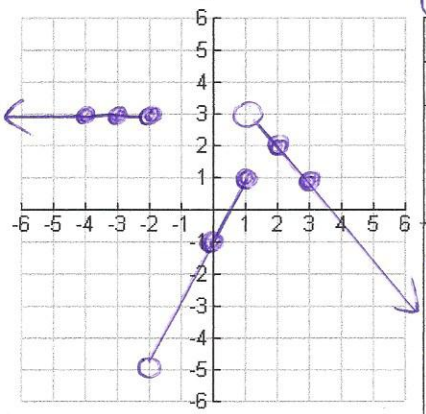


Table of Values		
3	$2x-1$	$4-x$
$(-2,3)$	$(-2,-5)$	$(1,3)$
	0	0
$(-3,3)$	$(0,-1)$	$(2,2)$
$(-4,3)$	$(1,1)$	$(3,1)$

d.)
$$f(x)=\begin{cases}-(x+2)^2+4 & \text{if } x<-1\cup x\ne -2\\ 1 & \text{if } x=-2\\ 2\sqrt{x+1}-2 & \text{if } x>-1\end{cases}$$

$(x<-1 \cup x\ne -2)\ (x=-2)\ (x>-1)$

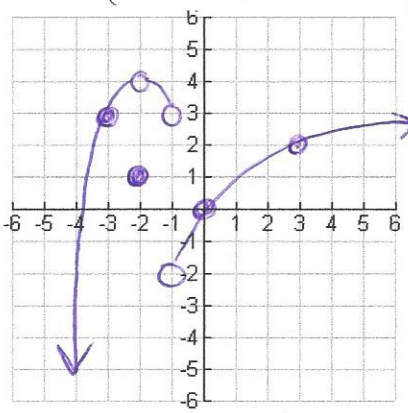


Table of Values		
$-(x+2)^2+4$	1	$2\sqrt{x+1}-2$
$(-1,3)$	$(-2,1)$	$(-1,-2)$
0		0
$(-2,4)$		$(0,0)$
0		
$(-3,3)$		$(3,2)$

Example 3: Complete the problem.

During a particular year, the taxes owed by a married person filing separately with an adjusted gross income of x dollars is given by the piecewise function below:

$$T(x)=\begin{cases}0.15x & \text{if } 0\le x<17{,}900\\ 0.28(x-17{,}900)+2685 & \text{if } 17{,}900\le x<43{,}250\\ 0.31(x-43{,}250)+9783 & \text{if } x\ge 43{,}250\end{cases}$$

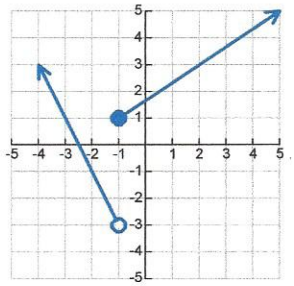
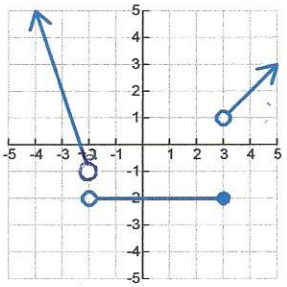
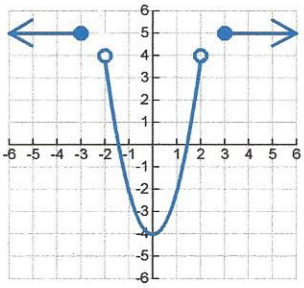
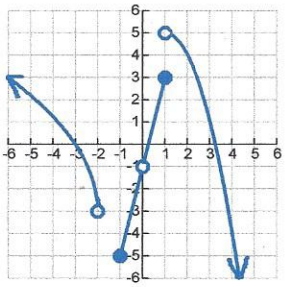
Find and interpret: T (70,000) + T (40,000)

$T(70{,}000)=.31(70000-43250)+9783=18075.50$

$T(40{,}000)=.28(40000-17900)+2685=8873$

$18075.50+8873=\$26{,}948.50$ represents how much taxes a married couple owes

Example 4: Determine the domain and range of each piecewise graph in interval notation.

Example 4a	Example 4b	Example 4c	Example 4d
			
D: $(-\infty,\infty)$	D: $(-\infty,-2)\cup(-2,\infty)$	D: $(-\infty,-3]\cup(-2,2)\cup[3,\infty)$	D: $(-\infty,-2)\cup[-1,0)\cup(0,\infty)$
R: $(-3,\infty)$	R: $[-2]\cup(-1,\infty)$	R: $[-4,4)\cup[5]$	R: $(-\infty,5)$