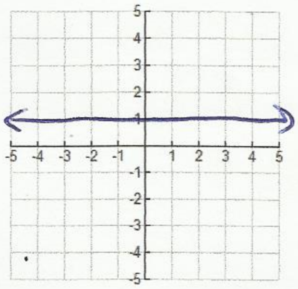
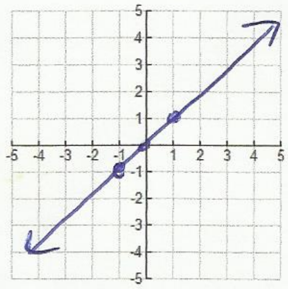
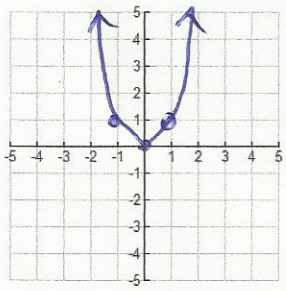
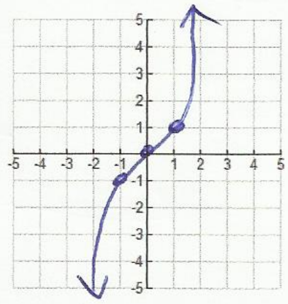
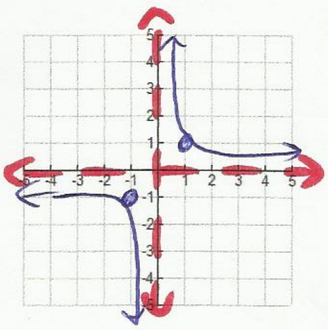
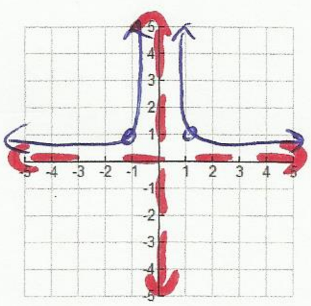
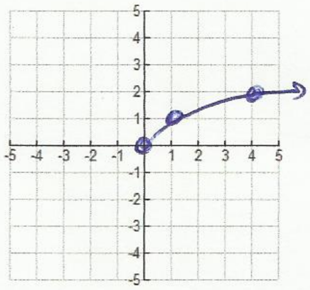
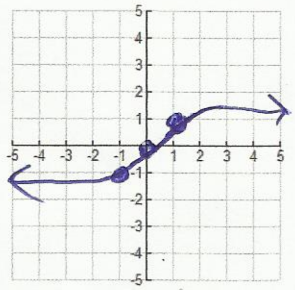
4.5 – Power Functions and Equations

Power Functions and Their Characteristics

– **power function** → a function in the form $y = k \cdot x^p$ (k and p are constants) where k is called the constant of proportionality and p is the exponent (p can be any real value, rational/irrational, or positive/negative)

Below are some examples of power functions where k = 1 and various values for p:

Power Function # 1	Power Function # 2	Power Function # 3	Power Function # 4
$y = x^0 \rightarrow y = 1$	$y = x^1 \rightarrow y = x$	$y = x^2$ (or x^4, x^6, …)	$y = x^3$ (or x^5, x^7, …)
			
Power Function # 5	**Power Function # 6**	**Power Function # 7**	**Power Function # 8**
$y = x^{-1}$ or $y = 1/x$	$y = x^{-2}$ or $y = 1/x^2$	$y = x^{1/2}$ or $y = \sqrt{x}$	$y = x^{1/3}$ or $y = \sqrt[3]{x}$
			

Example 1: Determine which are power functions, circle YES or NO. If YES, state value of k and p.

a.) $f(x) = 5\sqrt[3]{x^{12}}$ power function? Circle one: YES NO where k = 5 and p = 4

$5x^{\frac{12}{3}} = 5x^4$

b.) $f(x) = 6(x-1)^2$ power function? Circle one: YES NO where k = NA and p = NA

c.) $f(x) = \sqrt{\dfrac{36}{x^5}}$ power function? Circle one: YES NO where k = 6 and p = $-\frac{5}{2}$

$\sqrt{36x^{-5}} = 6x^{-\frac{5}{2}}$

d.) $10y + 2 = 5x^4 + 2$ power function? Circle one: YES NO where k = $\frac{1}{2}$ and p = 4

$\frac{10y}{10} = \frac{5x^4}{10} = \frac{1}{2}x^4$

e.) $f(x) = -5 \cdot 2^x$ power function? Circle one: YES NO where k = NA and p = NA

f.) $\frac{1}{4}y = (x-3)(x+3) + 9$ power function? Circle one: YES NO where k = 4 and p = 2

$\frac{1}{4}y = x^2 - 9 + 9 \rightarrow 4 \cdot \frac{1}{4}y = x^2 \cdot 4$

$y = 4x^2$

g.) $y + 3 = 3(x+1)$ power function? Circle one: YES NO where k = 3 and p = 1

$y + 3 = 3x + 3$

$y = 3x^1$

Example 2: Find the equation of a power function with the given information.

<table>
<tr><td colspan="3">Power Function in the form $y = k \cdot x^p$ where the point (x , y) and the point (1 , ?) (k = ?) are on the graph</td></tr>
<tr>
<td>a.) pts (2 , 12) ; (1 , 4) $k=4$
$y=k\cdot x^p$
$y=4\cdot x^p \rightarrow p=?$
$\frac{12}{4}=\frac{4\cdot 2^p}{4}$
$3=2^p$
$\frac{\log 3}{\log 2}=\frac{p\log 2}{\log 2} \rightarrow p=1.585$
$\boxed{y=4x^{1.585}}$</td>
<td>b.) pts (7 , 9) ; (1 , ½) $k=\frac{1}{2}$
$y=k\cdot x^p$
$y=\frac{1}{2}\cdot x^p \rightarrow p=?$
$2\cdot 9=\frac{1}{2}\cdot 7^p\cdot 2$
$18=7^p$
$\frac{\log 18}{\log 7}=\frac{p\log 7}{\log 7} \rightarrow p=1.485$
$\boxed{y=\frac{1}{2}x^{1.485}}$</td>
<td>c.) pts (4 , 0.375) and (9 , 0.25)
① $k=?$ ② $p=?$
$\frac{.375}{4^p}=\frac{k\cdot 4^p}{4^p}$
$k=\frac{.375}{4^p}$
$.25=k\cdot 9^p$
$.25=\left(\frac{.375}{4^p}\right)\cdot 9^p$
$\frac{.25}{.375}=\frac{.375\left(\frac{9}{4}\right)^p}{.375}$
$\frac{2}{3}=\left(\frac{9}{4}\right)^p$
$\frac{\log\left(\frac{2}{3}\right)}{\log\left(\frac{9}{4}\right)}=\frac{p\log\left(\frac{9}{4}\right)}{\log\left(\frac{9}{4}\right)}$
$p=-.5$ or $-\frac{1}{2}$
$k=\frac{.375}{4^{-.5}}$
$k=.75$ or $\frac{3}{4}$
$\boxed{y=\frac{3}{4}x^{-\frac{1}{2}}}$</td>
</tr>
</table>

Solving Power Equations – Direct and Inverse Variations

Direct Variation Equation: $y=k\cdot x^p$ (where p is a positive #)	**Inverse Variation Equation:** $y=k\cdot x^{-p}$ or $y=\frac{k}{x^p}$ (where p is a negative #)

Example 3: Complete each variation problem.

<table>
<tr>
<td>a.) Suppose y is directly proportional to x. If y = 18 when x = 8, find the constant of proportionality (k). After finding the formula for y, then use it to find x when y = 27. (understood exponent of 1 since no mention of anything else)
$y=k\cdot x^1$
$\frac{18}{8}=\frac{k\cdot 8}{8}$
$k=2.25$
① $\boxed{y=2.25x}$
② $\frac{27}{2.25}=\frac{2.25x}{2.25}$
$\boxed{x=12}$</td>
<td>b.) Suppose c is inversely proportional to the square of d. If c = 4 when d = 2, find the constant of proportionality (k). After finding the formula for c, then use it to find c when d = – 8.
$c=\frac{k}{d^2}$
$4=\frac{k}{(2^2)}$
$4=\frac{k}{4}$
$k=16$
① $\boxed{c=\frac{16}{d^2}}$
② $c=\frac{16}{(-8)^2}$
$c=\frac{16}{64}$
$\boxed{c=\frac{1}{4}}$</td>
</tr>
<tr>
<td colspan="2">c.) The radius of a sphere is directly proportional to the cube root of its volume. If a sphere of radius 18.2 cm has a volume of 25,252.4 cm³, what is the volume of a sphere if the radius is 19.3 cm?
$r=k\sqrt[3]{V}$
$18.2=k\sqrt[3]{25252.4}$
$\frac{18.2}{29.338}=\frac{29.338k}{29.338}$
$k=.62$
$r=.62\sqrt[3]{V}$
$\frac{19.3}{6.2}=\frac{6.2\sqrt[3]{V}}{6.2}$
$(31.129)^3=\left(\sqrt[3]{V}\right)^3$
$\boxed{V=30,164.45\text{cm}^3}$</td>
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