

4.4 – Exponential/Logarithmic Word Problems

Exp Growth	Exp Decay	Comp'd With n values	Comp'd Contin'ly						
$A = a(1 + r)^t$ <p>A = final amount a = initial amount r = rate of growth t = time</p> <p><u>Key words:</u> Increase Grows Appreciates</p>	$A = a(1 - r)^t$ <p>A = final amount a = initial amount r = rate of decay t = time</p> <p><u>Key words:</u> Decrease Decays Depreciates</p>	$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$ <p>A = final amount P = principle amount r = interest rate n = # of times \$ is comp'd t = time (always in years)</p> <table><tr><td>annually: n = 1</td><td>monthly: n = 12</td></tr><tr><td>semiannually: n = 2</td><td>weekly: n = 52</td></tr><tr><td>quarterly: n = 4</td><td>daily: n = 365</td></tr></table>	annually: n = 1	monthly: n = 12	semiannually: n = 2	weekly: n = 52	quarterly: n = 4	daily: n = 365	$A = Pe^{r \cdot t}$ <p>A = final amount P = principle amount r = interest rate e = exp function</p> <p><u>Key word:</u> Continuously</p>
annually: n = 1	monthly: n = 12								
semiannually: n = 2	weekly: n = 52								
quarterly: n = 4	daily: n = 365								

Example 1: Complete each exponential word problem.

a.) You bought a car for \$24,000. The car's value has <u>depreciated</u> by 8.7% each year. How much will your car be worth 11 years from initially buying it? <i>exp decay</i> $a = 24000$ $r = .087$ $t = 11$ $A = ?$ $A = 24000(1 - .087)^{11}$ $= \boxed{\$8,818.40}$	b.) In 1910, the population of a city was 120,000. Since then, the population has <u>increased</u> by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in 2014? <i>exp growth</i> $a = 120,000$ $r = .015$ $t = 2014 - 1910 = 104$ $A = ?$ $A = 120,000(1 + .015)^{104}$ $= \boxed{564,481 \text{ people}}$
c.) An island initially had 500 rabbits and is <u>growing</u> each year. After 16 years, there are 45,000 rabbits that inhabit the island. What is the growth rate of the rabbits on the island? <i>exp growth</i> $a = 500$ $A = 45000$ $t = 16$ $r = ?$ $45000 = 500(1 + r)^{16}$ $\frac{45000}{500} = \frac{500(1 + r)^{16}}{500}$ $90 = (1 + r)^{16}$ $\sqrt[16]{90} = \sqrt[16]{(1 + r)^{16}}$ $1.324769 = 1 + r$ $r = .324769$ $\boxed{\text{rate} = 32.5\%}$	d.) Amber has a savings account in which her money is being compounded continuously with a 3% interest rate. After 8 years, Amber's account has a balance of \$1,907. What was Amber's initial deposit for the account? $A = 1907$ $P = ?$ $r = .03$ $t = 8$ $1907 = Pe^{.03(8)}$ $1907 = Pe^{.24}$ $\frac{1907}{e^{.24}} = \frac{Pe^{.24}}{e^{.24}}$ $\boxed{P = \$1500.10}$
e.) Mike decides to invest \$400 into an account that has a 6% interest rate. i.) What is the balance in the account after 4 years if the account is being compounded monthly? $A = ?$ $P = 400$ $r = .06$ $n = 12$ $t = 4$ $A = 400\left(1 + \frac{.06}{12}\right)^{48}$ $= \boxed{\$508.20}$ ii.) What is the balance in the account after 4 years if the account is being compounded continuously? $A = ?$ $P = 400$ $r = .06$ $t = 4$ $A = 400e^{.06(4)}$ $= \boxed{\$508.50}$	f.) Desmond is investing \$800 into an account with a 5% interest rate. i.) How long will it take for the account to be \$2800 if the money is compounded quarterly? $A = 2800$ $P = 800$ $r = .05$ $n = 4$ $t = ?$ $2800 = 800\left(1 + \frac{.05}{4}\right)^{4t}$ $3.5 = (1.0125)^{4t}$ $\log 3.5 = \log (1.0125)^{4t}$ $\log 3.5 = 4t \log (1.0125)$ $\frac{\log 3.5}{4 \log (1.0125)} = \frac{4t \log (1.0125)}{4 \log (1.0125)}$ $\boxed{t = 25.24 \text{ yr}}$ ii.) How long will it take for the account to double if the money is compounded continuously? $A = 800 \times 2 = 1600$ $P = 800$ $r = .05$ $t = ?$ $1600 = 800e^{.05t}$ $\frac{1600}{800} = \frac{800e^{.05t}}{800}$ $2 = e^{.05t}$ $\ln 2 = \ln e^{.05t}$ $\ln 2 = .05t$ $\frac{\ln 2}{.05} = \frac{.05t}{.05}$ $\boxed{t = 13.94 \text{ yr}}$

Log Scale – pH Scale	Log Scale – Decibel Scale	Log Scale – Memory Recall
$pH = -\log(H)$ <p>pH = acidity of a solution If pH < 7 then solution is acidic If pH = 7 then solution is neutral If pH > 7 then solution is basic H = hydrogen ions in M where H has to be in scient. not.</p>	$D = 10(\log I + 12)$ <p>D = intensity level in dB (decibels) I = intensity of any given sound where measures in W/m^2 (Watts/meters²)</p>	$R = 75 - [6\ln(t+1)]$ <p>R = percent of the info retained t = number of months that have gone by after being presented with info.</p>

Example 2: Complete each logarithmic word problem.

<p>a.) The hydrogen ion of a sample of human blood was measured to be $H = 3.16 \times 10^{-8}$ M. Find the pH and classify the sample.</p> $pH = -\log(3.16 \times 10^{-8})$ $pH = 7.5 \text{ (basic)}$	<p>c.) A jet engine during takeoff has an intensity measured at $100 W/m^2$. What is the jet engine's intensity level? $D = ?$</p> $D = 10(\log(100) + 12)$ $D = 140 \text{ dB}$	<p>e.) What percent of memory was retained <u>6 months</u> after being presented the information? $R = ?$</p> $R = 75 - [6\ln(6+1)]$ $R = 75 - [6\ln(7)]$ $R = 63.3\%$
<p>b.) The most acidic rainfall ever measured occurred in Scotland in 1974, its pH = 3.8. What is the hydrogen ion concentration of this rainfall?</p> $\frac{3.8}{-1} = \frac{-\log H}{-1}$ $-3.8 = \log_{10} H = -3.8$ $H = 10^{-3.8}$ $H = 1.6 \times 10^{-4} \text{ M}$	<p>d.) The intensity level of sound of a subway train was measured to be 98 dB. What is the intensity? $D = 98$</p> $I = ?$ $\frac{98}{10} = \frac{10(\log I + 12)}{10}$ $9.8 = \log I + 12$ $\frac{-12}{-12} = \frac{-12}{-12}$ $-2.2 = \log_{10} I = -2.2$ $I = 10^{-2.2}$ $I = .0063$ $I = 6.3 \times 10^{-3} W/m^2$	<p>f.) After <u>how many months</u> did the average person retain only half of the presented information? $R = 50$</p> $50 = 75 - [6\ln(t+1)]$ $-75 + 75$ $-25 = -[6\ln(t+1)]$ $\frac{25}{6} = \frac{6\ln(t+1)}{6}$ $4.16667 = \ln(t+1)$ $e^{4.16667} = t+1$ $\frac{-1}{-1}$ $t = 63.5 \text{ months}$