4.3 – Exponential and Logarithmic Equations

Solving Exponential Equations Examples:

Notes: 1.) Keep answers as (reduced) fractions when possible.
If a decimal can NOT be turned into a fraction – ROUND to 3 decimal places

Exponential Equations: Type 1 – Both Sides have the Same Base

1a.) $2^{2x+15}=8$

$$2^{2x+15}=2^3$$
$$2x+15=3$$
$$-15 \quad -15$$
$$\frac{2x}{2}=\frac{-12}{2}$$
$$\boxed{x=-6}$$

1b.) $4^{3x}=32^{x+1}$

$$(2^2)^{3x}=(2^5)^{x+1}$$
$$2\cdot 3x=5(x+1)$$
$$6x=5x+5$$
$$-5x \quad -5x$$
$$\boxed{x=5}$$

1c.) $3^{2x+5}=\left(\frac{1}{9}\right)^{x-1}$

$$3^{2x+5}=(3^{-2})^{x-1}$$
$$2x+5=-2(x-1)$$
$$2x+\cancel{5}=-\cancel{2x}+2$$
$$+2x \quad -\cancel{5} \quad +\cancel{2x} \quad -5$$
$$\frac{4x}{4}=\frac{-3}{4}$$
$$\boxed{x=\frac{-3}{4}}$$

1d.) $25\cdot\left(\frac{1}{125}\right)^{x+4}=\sqrt{3125}$

$$5^2\cdot(5^{-3})^{x+4}=(5^5)^{\frac{1}{2}}$$
$$2-3(x+4)=\frac{5}{2}$$
$$2-3x-12=\frac{5}{2}$$
$$2(-3x-10)=\frac{5}{\cancel{2}}\cancel{\cdot 2}$$
$$-6x-20=5$$
$$\frac{-6x}{-6}=\frac{25}{-6}$$
$$\boxed{x=\frac{25}{-6}}$$

Exponential Equations: Type 2: Both Sides are NOT the Same Base – Take log (or ln) of both sides

1e.) $3^{2x-1}=7$

$$\log 3^{2x-1}=\log 7$$
$$\frac{(2x-1)\cancel{\log 3}}{\cancel{\log 3}}=\frac{\log 7}{\log 3}$$
$$2x-1=1.771244$$
$$+1 \quad +1$$
$$\frac{2x}{2}=\frac{2.771244}{2}$$
$$\boxed{x=1.386}$$

1f.) $3\cdot 4^x+11=2$

$$-11 \quad -11$$
$$\frac{3\cdot 4^x}{3}=\frac{-9}{3}$$
$$4^x=-3$$
$$\log 4^x=\log(-3)$$
$$\frac{x\log 4}{\log 4}=\frac{\log(-3)}{\log 4}$$

can't do!

$$\boxed{\text{no solution } \varnothing}$$

1g.) $\frac{2e^{5x-3}}{2}=\frac{16}{2}$

$$e^{5x-3}=8$$
$$\cancel{\ln} e^{5x-3}=\ln 8$$
$$5x-3=\ln 8$$
$$+3 \quad +3$$
$$\frac{5x}{5}=\frac{(\ln(8)+3)}{5}$$
$$\boxed{x=1.016}$$

1h.) $\frac{12}{1+e^{-x}} \times \frac{2}{1}$

$$\frac{2(1+e^{-x})}{2}=\frac{12}{2}$$
$$1+e^{-x}=6$$
$$-1 \quad -1$$
$$e^{-x}=5$$
$$\cancel{\ln} e^{-x}=\ln 5$$
$$\frac{-x}{-1}=\frac{\ln(5)}{-1}$$
$$\boxed{x=-1.609}$$

Solving Logarithmic Equations Examples:

Notes: 1.) Keep answers as (reduced) fractions when possible – if not, ROUND to 3 places!

2.) Remember – You CAN NOT take log of a negative number or zero (b/c VA: x = 0)
Some solutions MAY OR MAY NOT WORK (could also have no solutions)!
When done solving a log equation – Check to see that it works in the original problem!

Logarithmic Equations: Type 1 – Both sides (and every term) is a logarithm

2a.) $\cancel{\log_5}(6-3x)=\cancel{\log_5}(10-5x)$ $5 \qquad 5$ $\cancel{6}-3x=10-\cancel{5x}$ $\cancel{-6}\ +5x \quad -6\ +\cancel{5x}$ $\frac{2x}{2}=\frac{4}{2}$ $x=2$ (will get log(0)) $\boxed{\text{no solution}}$	2b.) $\log(x-3)+\log x=\log 28$ $10^{\log(x-3)\cdot x}=10^{\log 28}$ $x^2-3x=28$ $x^2-3x-28=0$ $(x+4)(x-7)=0$ $x+4=0 \quad x-7=0$ $x=-4$ (will get log(neg#)) $\quad \boxed{x=7}$ only answer	2c.) $\ln(2x+1)-\ln(x-1)=\ln 7$ $e^{\ln\left(\frac{2x+1}{x-1}\right)}=e^{\ln 7}$ $\frac{2x+1}{x-1}=\frac{7}{1}$ $7x-\cancel{7}=2\cancel{x}+1$ $-2x\ \cancel{+7} \quad -\cancel{2x}\ +7$ $\frac{5x}{5}=\frac{8}{5}$ $\boxed{x=\frac{8}{5}}$

Logarithmic Equations: Type 2 – SINGLE LOG on one side and a CONSTANT on the other side

2d.) $\log_3(2x+15)=2$ $3^2=2x+15$ $9=2x+15$ $-15 \qquad -15$ $\frac{-6}{2}=\frac{2x}{2}$ $\boxed{x=-3}$	2e.) $\log_2 x+\log_2(x-2)=3$ $\log_2 x(x-2)=3$ $2^3=x^2-2x$ $8=x^2-2x$ $x^2-2x-8=0$ $(x+2)(x-4)=0$ $x+2=0 \quad x-4=0$ $x=-2$ (will get log(neg#)) $\quad \boxed{x=4}$ only answer	2f.) $4-6\ln(3x-1)=-20$ $-4 \qquad -4$ $\frac{-6\ln(3x-1)}{-6}=\frac{-24}{-6}$ $e^{\cancel{\ln}(3x-1)}=e^4$ $3x-1=e^4$ $+1 \quad +1$ $\frac{3x}{3}=\frac{(e^4+1)}{3}$ $\boxed{x=18.533}$