4.2 – Exponential and Logarithmic Functions

Exponential / Logarithmic Functions and Their Characteristics Exp and Log are inverses

– **exponential function** → a function in the form $y=(b)^x$ where $b > 0$, $b \neq 1$, and x is IR.

Exponential Function's Characteristics	
Domain: $(-\infty, \infty)$	Range: $(0, \infty)$
Common Pt: $(0, 1)$	Asymptote: HA: $y = 0$

Transforming Exp Graph: $y = b^{(x \pm c)} \pm d$

a.) # is on "outside" → + d: up – d: down

b.) # is on "inside" → + c: left – c: right

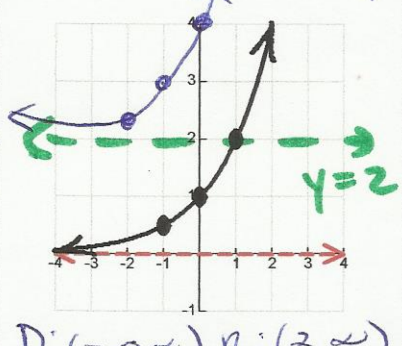
c.) Domain of Transform Graph: $(-\infty, \infty)$

d.) Range of Transform Graph: (HA #, ∞)

Graphs of Exponential Functions

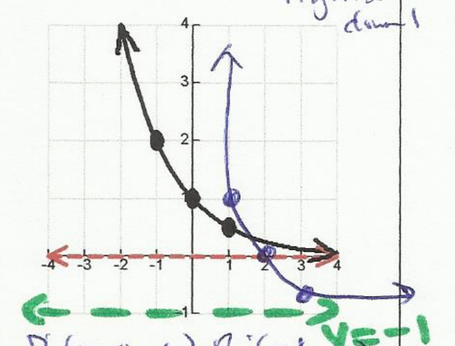
a.) Graph of $y = (2)^x$

Graph: $y = (2)^{x+1} + 2$ left 1, up 2



b.) Graph of $y = (½)^x$

Graph: $y = (½)^{x-2} - 1$ right 2, down 1



– **logarithmic function** → a function in the form $y=\log_b(x)$ where $b > 0$, $b \neq 1$, and x is IR.

Logarithmic Function's Characteristics	
Domain: $(0, \infty)$	Range: $(-\infty, \infty)$
Common Pt: $(1, 0)$	Asymptote: VA: $x = 0$

Transforming Log Graph: $y = \log_b(x \pm c) \pm d$

a.) # is on "outside" → + d: up – d: down

b.) # is on "inside" → + c: left – c: right

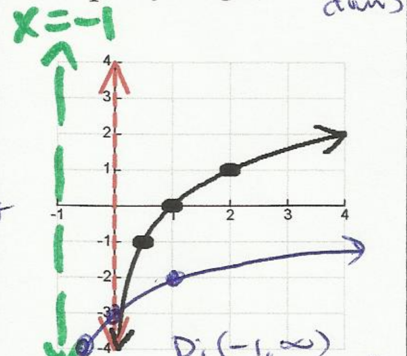
c.) Domain of Transform Graph: (VA #, ∞)

d.) Range of Transform Graph: $(-\infty, \infty)$

Graphs of Logarithmic Functions

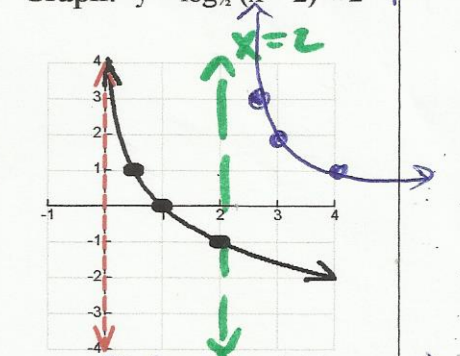
a.) Graph of $y = \log_2(x)$

Graph: $y = \log_2(x + 1) - 3$ left 1, down 3



b.) Graph of $y = \log_{½}(x)$

Graph: $y = \log_{½}(x - 2) + 2$ right 2, up 2



Example 1: **State the asymptote, domain, and range of each given function using interval notation.**

Given Exp / Log Function	Asymptote	Domain	Range
a.) $f(x) = 4^{x-3} + 5$ right 3, up 5 — Exp Funct	HA: $y = 5$	$(-\infty, \infty)$	$(5, \infty)$
b.) $f(x) = \log_3(x + 4) - 3$ left 4, down 3 — Log Funct	VA: $x = -4$	$(-4, \infty)$	$(-\infty, \infty)$
c.) $f(x) = (⅓)^{x+5} - 2$ left 5, down 2 — Exp Funct	HA: $y = -2$	$(-\infty, \infty)$	$(-2, \infty)$
d.) $f(x) = \ln(x - 4) + 1$ right 4, up 1 — Log Funct	VA: $x = 4$	$(4, \infty)$	$(-\infty, \infty)$

Properties of Logarithmic Functions

Basic Log Property (Hamburger Helper Hand) → helps to convert from LOG form to EXP FORM

Logarithmic Form		**Exponential Form**
$\log_b y = x$	⟵⟶	$b^x = y$

Example 2: Convert

a.) $\log_2 8 = 3 \leftrightarrow$ $2^3 = 8$

b.) $\log_5 625 = 4$ $\leftrightarrow 5^4 = 625$

Laws of Logarithms →

Law # 1: $\log_b X + \log_b Y = \log_b(X \cdot Y)$

Law # 2: $\log_b X - \log_b Y = \log_b\left(\frac{X}{Y}\right)$

Law # 3: $\log_b X^Y = Y \cdot \log_b X$

Example 3: Evaluate each expression or find the value of x.

a.) $\log_3 9 = x$ $3^x = 9$ $3^x = 3^2$ $\boxed{x=2}$	b.) $\log_4 8 = x$ $4^x = 8$ $(2^2)^x = 2^3$ $2x = 3$ $\boxed{x = \frac{3}{2}}$	c.) $\log_2\left(\frac{1}{16}\right) = x$ $2^x = \frac{1}{16}$ $2^x = 2^{-4}$ $\boxed{x = -4}$	d.) $\log_8\left(\frac{1}{256}\right) = x$ $8^x = \frac{1}{256}$ $(2^3)^x = 2^{-8}$ $3x = -8$ $\boxed{x = -\frac{8}{3}}$
e.) $\log_{36}\sqrt{6} = x$ $36^x = \sqrt{6}$ $(6^2)^x = 6^{\frac{1}{2}}$ $\frac{2x}{2} = \frac{\frac{1}{2}}{2}$ $\boxed{x = \frac{1}{4}}$	f.) $\log_x 5 = \frac{1}{3}$ $x^{\frac{1}{3}} = 5$ $(\sqrt[3]{x})^3 = (5)^3$ $\boxed{x = 125}$	g.) $\log(100)^4$ $\log_{10}(10^2)^4$ $\log_{10} 10^8 = x$ $10^x = 10^8$ $\boxed{x = 8}$	h.) $\ln\left(\frac{1}{e^3}\right)$ $\ln(e^{-3})$ $= \boxed{-3}$
i.) $\log_2 112 - \log_2 7$ $\log_2\left(\frac{112}{7}\right)$ $\log_2 16 = x$ $2^x = 16$ $2^x = 2^4$ $\boxed{x = 4}$	j.) $\log_{12} 9 + \log_{12} 16$ $\log_{12}(9 \cdot 16)$ $\log_{12} 144 = x$ $12^x = 144$ $12^x = 12^2$ $\boxed{x = 2}$	k.) $e^{3\ln 2 - \ln 4}$ $e^{\ln 2^3 - \ln 4}$ $e^{\ln 8 - \ln 4}$ $e^{\ln\left(\frac{8}{4}\right)}$ $\cancel{e^{\ln}} 2$ $= \boxed{2}$	l.) $\log_{10}\sqrt{\frac{1}{10}} = x$ $10^x = (10^{-1})^{\frac{1}{2}}$ $10^x = 10^{-\frac{1}{2}}$ $\boxed{x = \frac{-1}{2}}$