2.6 – Binomial Probability (Experiments)

– **binomial probability (experiments)** → used to find probabilities where there are 2 possible outcomes where key words to look for to know when to use this technique are Exactly, At most, At least

Ex: What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice (A – D possible answer choices) quiz if you guess at every question? →

$P(4 \text{ exactly right}) = {}_5C_4 \cdot \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^1 = 1.5\%$

(4 way to get correct answer; add up to...; 4 correct; 1 wrong)

Binomial Exp "Formula" → $_{\text{total trials}}C_{\text{successes}} \cdot (\text{successes prob})^{\text{success power}} \cdot (\text{failure prob})^{\text{failures power}}$

Examples: Find each probability using the Binomial Experiment "Formula".

1.) If a family has 4 children, what is the probability that they have exactly 3 boys?

$P(\text{exactly 3 boys}) = {}_4C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^1 = 25\%$

2.) Suppose that a coin is tossed 5 times, what is the probability of getting exactly 2 heads?

$P(\text{exactly 2 heads}) = {}_5C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 = 31.3\%$

3.) A die is rolled 3 times, what is the probability of getting exactly three 5's?

$P(\text{exactly three 5's}) = {}_3C_3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^0 = .46\%$

4.) Tarin and Sam are playing a certain board game, the probability of Tarin winning a game is 75%. If they play 5 games, then what is the probability that Sam will win exactly 3 games?

Sam winning = 25% (since Tarin is 75%)

$P(\text{Sam winning exactly 3 games}) = {}_5C_3 \cdot (.25)^3 (.75)^2 = 8.8\%$

5.) Suppose that when hockey star Jamarie Jones takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game one night.

a.) What is the probability that he will score exactly 1 goal?

$P(\text{exactly 1 goal}) = {}_6C_1 \cdot \left(\frac{1}{7}\right)^1 \left(\frac{6}{7}\right)^5 = 39.7\%$

b.) What is the probability that he will score at most 2 goals?

$P(0) + P(1) + P(2) \rightarrow$

$= {}_6C_0 \left(\frac{1}{7}\right)^0 \left(\frac{6}{7}\right)^6 + {}_6C_1 \left(\frac{1}{7}\right)^1 \left(\frac{6}{7}\right)^5 + {}_6C_2 \left(\frac{1}{7}\right)^2 \left(\frac{6}{7}\right)^4 = 95.8\%$

c.) What is the probability that he will score at least 4 goals?

$P(4) + P(5) + P(6) \rightarrow$

$= {}_6C_4 \left(\frac{1}{7}\right)^4 \left(\frac{6}{7}\right)^2 + {}_6C_5 \left(\frac{1}{7}\right)^5 \left(\frac{6}{7}\right)^1 + {}_6C_6 \left(\frac{1}{7}\right)^6 \left(\frac{6}{7}\right)^0 = .49\%$