

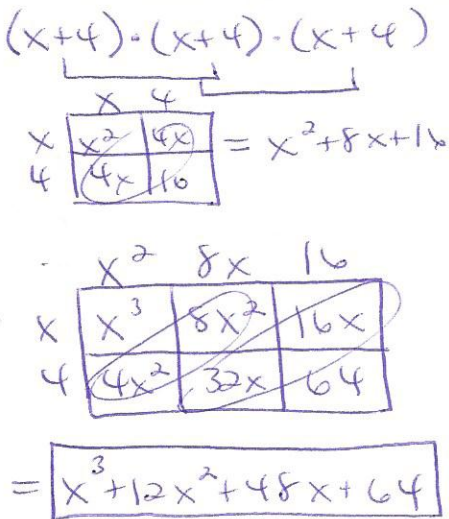
2.5 – Binomial Expansions (Using Combinations)

- **binomial** → a two term expression which include one or more variables
- **binomial expansion** → a binomial raised to nth power such as $(a + b)^n$, where n is a whole number, where the coefficients are symmetric, the exponent a decreases and the exponent b increases and the sum of the exponents in each term is n

▪ There are three ways to show a binomial expansion:

- 1.) Box Method → Organize way to expand a binomial but can be time consuming.
- 2.) Pascal's Triangle → Systemic way to expand a binomial and contains multiple patterns
- 3.) Combinations → Condensed way to expand a binomial and contains easier patterns

▪ Let's look at a simple example: $(x + 4)^3$ → Produce the answer through the 3 different methods:

Box Method	Pascal's Triangle	Combinations
$(x+4) \cdot (x+4) \cdot (x+4)$ 	$ \begin{array}{cccc} & & 1 & \longrightarrow (x+4)^0 \\ & & 1 & 1 \longrightarrow (x+4)^1 \\ & 1 & 2 & 1 \longrightarrow (x+4)^2 \\ 1 & 3 & 3 & 1 \longrightarrow (x+4)^3 \end{array} $ $ \begin{aligned} &= 1(x)^3(4)^0 + 3(x)^2(4)^1 \\ &+ 3(x)^1(4)^2 + 1(x)^0(4)^3 \\ &= \boxed{x^3 + 12x^2 + 48x + 64} \end{aligned} $	$ \begin{aligned} &{}^3C_0 (x)^3 (4)^0 = 1 \cdot x^3 \cdot 1 \\ &{}^3C_1 (x)^2 (4)^1 = 3 \cdot x^2 \cdot 4 \\ &{}^3C_2 (x)^1 (4)^2 = 3 \cdot x \cdot 16 \\ &{}^3C_3 (x)^0 (4)^3 = 1 \cdot 1 \cdot 64 \\ &= \boxed{x^3 + 12x^2 + 48x + 64} \end{aligned} $

Examples: Complete each problem below by expanding the binomial through combinations.

<p>1.) Expand: $(2x + 5)^4$</p> $ \begin{aligned} &{}^4C_0 (2x)^4 (5)^0 = 1 \cdot 16 \cdot x^4 \cdot 1 \\ &{}^4C_1 (2x)^3 (5)^1 = 4 \cdot 8 \cdot x^3 \cdot 5 \\ &{}^4C_2 (2x)^2 (5)^2 = 6 \cdot 4 \cdot x^2 \cdot 25 \\ &{}^4C_3 (2x)^1 (5)^3 = 4 \cdot 2 \cdot x \cdot 125 \\ &{}^4C_4 (2x)^0 (5)^4 = 1 \cdot 1 \cdot 625 \\ &= \boxed{16x^4 + 160x^3 + 600x^2 + 1000x + 625} \end{aligned} $	<p>2.) Find 3rd term: $(x^3 - 2)^7$</p> $ \begin{aligned} &{}^7C_0 (x^3)^7 (-2)^0 \\ &{}^7C_1 (x^3)^6 (-2)^1 \\ &\textcircled{{}^7C_2 (x^3)^5 (-2)^2} \rightarrow \text{3rd term} \\ &\quad \downarrow \\ &= 21 \cdot x^{15} \cdot 4 \\ &= \boxed{84x^{15}} \end{aligned} $	<p>3.) Find <u>middle</u> term: $(3x^2 - 4y^4)^{10}$</p> $ \begin{aligned} &\frac{10}{2} = 5 \\ &{}^{10}C_5 (3x^2)^5 (-4y^4)^5 \\ &= 252 \cdot 243x^{10} \cdot (-1024y^{20}) \\ &= \boxed{-62705664x^{10}y^{20}} \end{aligned} $
---	--	--