2.5 – Binomial Expansions (Using Combinations)

– **binomial** → a two term expression which include one or more variables

– **binomial expansion** → a binomial raised to nth power such as $(a + b)^n$, where n is a whole number, where the coefficients are symmetric, the exponent a decreases and the exponent b increases and the sum of the exponents in each term is n

- There are three ways to show a binomial expansion:
  1.) Box Method → Organize way to expand a binomial but can be time consuming.
  2.) Pascal's Triangle → Systemic way to expand a binomial and contains multiple patterns
  3.) Combinations → Condensed way to expand a binomial and contains easier patterns

- Let's look at a simple example: $(x + 4)^3$ → Produce the answer through the 3 different methods:

Box Method	Pascal's Triangle	Combinations
$(x+4)\cdot(x+4)\cdot(x+4)$ $\begin{array}{c\|cc} & x & 4 \\ \hline x & x^2 & 4x \\ 4 & 4x & 16 \end{array} = x^2+8x+16$ $\begin{array}{c\|ccc} & x^2 & 8x & 16 \\ \hline x & x^3 & 8x^2 & 16x \\ 4 & 4x^2 & 32x & 64 \end{array}$ $= \boxed{x^3+12x^2+48x+64}$	$1 \longrightarrow (x+4)^0$ $1 \quad 1 \longrightarrow (x+4)^1$ $1 \quad 2 \quad 1 \rightarrow (x+4)^2$ $1 \quad 3 \quad 3 \quad 1 \rightarrow (x+4)^3$ $= 1(x)^3(4)^0 + 3(x)^2(4)^1$ $+ 3(x)^1(4)^2 + 1(x)^0(4)^3$ $= \boxed{x^3+12x^2+48x+64}$	matches, exp ↓, exp ↑ ${}_3C_0(x)^3(4)^0 = 1\cdot x^3\cdot 1$ matches ${}_3C_1(x)^2(4)^1 = 3\cdot x^2\cdot 4$ ${}_3C_2(x)^1(4)^2 = 3\cdot x\cdot 16$ ${}_3C_3(x)^0(4)^3 = 1\cdot 1\cdot 64$ $= \boxed{x^3+12x^2+48x+64}$

Examples: Complete each problem below by expanding the binomial through combinations.

1.) Expand: $(2x + 5)^4$	2.) Find 3rd term: $(x^3 – 2)^7$	3.) Find middle term: $(3x^2 – 4y^4)^{10}$
${}_4C_0(2x)^4(5)^0 = 1\cdot 16\cdot x^4\cdot 1$ ${}_4C_1(2x)^3(5)^1 = 4\cdot 8\cdot x^3\cdot 5$ ${}_4C_2(2x)^2(5)^2 = 6\cdot 4\cdot x^2\cdot 25$ ${}_4C_3(2x)^1(5)^3 = 4\cdot 2x\cdot 125$ ${}_4C_4(2x)^0(5)^4 = 1\cdot 1\cdot 625$ $= \boxed{16x^4+160x^3+600x^2+1000x+625}$	${}_7C_0(x^3)^7(-2)^0$ ${}_7C_1(x^3)^6(-2)^1$ ${}_7C_2(x^3)^5(-2)^2 \rightarrow$ 3rd term $\downarrow$ $= 21\cdot x^{15}\cdot 4$ $= \boxed{84x^{15}}$	$\frac{10}{2} = 5$ ${}_{10}C_5(3x^2)^5(-4y^4)^5$ $= 252\cdot 243x^{10}(-1024y^{20})$ $= \boxed{-62705664x^{10}y^{20}}$