Advanced Functions – 2.5 WS: Binomial Expansions   Name: Key

I. Using combinations to expand each binomial completely. Must show work!

1.) $(m+5)^3$

$_3C_0\, m^3 5^0 = 1\cdot m^3\cdot 1$
$_3C_1\, m^2 5^1 = 3\cdot m^2\cdot 5$
$_3C_2\, m^1 5^2 = 3\, m\cdot 25$
$_3C_3\, m^0 5^3 = 1\cdot 1\cdot 125$
$= \boxed{m^3+15m^2+75m+125}$

2.) $(4x-y)^4$

$_4C_0\, (4x)^4(-y)^0 = 1\cdot 256x^4\cdot 1$
$_4C_1\, (4x)^3(-y)^1 = 4\cdot 64x^3\cdot -y$
$_4C_2\, (4x)^2(-y)^2 = 6\cdot 16x^2\cdot y^2$
$_4C_3\, (4x)^1(-y)^3 = 4\cdot 4x\cdot -y^3$
$_4C_4\, (4x)^0(-y)^4 = 1\cdot 1\cdot y^4$
$= \boxed{256x^4-256x^3y+96x^2y^2-16xy^3+y^4}$

3.) $(2x+3)^5$

$_5C_0\, (2x)^5(3)^0 = 1\cdot 32x^5\cdot 1$
$_5C_1\, (2x)^4(3)^1 = 5\cdot 16x^4\cdot 3$
$_5C_2\, (2x)^3(3)^2 = 10\cdot 8x^3\cdot 9$
$_5C_3\, (2x)^2(3)^3 = 10\cdot 4x^2\cdot 27$
$_5C_4\, (2x)^1(3)^4 = 5\cdot 2x\cdot 81$
$_5C_5\, (2x)^0(3)^5 = 1\cdot 1\cdot 243$
$= \boxed{32x^5+240x^4+720x^3+1080x^2+810x+243}$

4.) $(3x^3-4)^3$

$_3C_0\, (3x^3)^3(-4)^0 = 1\cdot 27x^9\cdot 1$
$_3C_1\, (3x^3)^2(-4)^1 = 3\cdot 9x^6\cdot -4$
$_3C_2\, (3x^3)^1(-4)^2 = 3\cdot 3x^3\cdot 16$
$_3C_3\, (3x^3)^0(-4)^3 = 1\cdot 1\cdot -64$
$= \boxed{27x^9-108x^6+144x^3-64}$

5.) $(2a+5b^2)^6$

$_6C_0\, (2a)^6(5b^2)^0 = 1\cdot 64a^6\cdot 1$
$_6C_1\, (2a)^5(5b^2)^1 = 6\cdot 32a^5\cdot 5b^2$
$_6C_2\, (2a)^4(5b^2)^2 = 15\cdot 16a^4\cdot 25b^4$
$_6C_3\, (2a)^3(5b^2)^3 = 20\cdot 8a^3\cdot 125b^6$
$_6C_4\, (2a)^2(5b^2)^4 = 15\cdot 4a^2\cdot 625b^8$
$_6C_5\, (2a)^1(5b^2)^5 = 6\cdot 2a\cdot 3125b^{10}$
$_6C_6\, (2a)^0(5b^2)^6 = 1\cdot 1\cdot 15625b^{12}$
$= \boxed{64a^6+960a^5b^2+6000a^4b^4+20000a^3b^6+37500a^2b^8+37500ab^{10}+15625b^{12}}$

6.) $(1-4n)^3$

$_3C_0\, (1)^3(-4n)^0 = 1\cdot 1\cdot 1$
$_3C_1\, (1)^2(-4n)^1 = 3\cdot 1\cdot -4n$
$_3C_2\, (1)^1(-4n)^2 = 3\cdot 1\cdot 16n^2$
$_3C_3\, (1)^0(-4n)^3 = 1\cdot 1\cdot -64n^3$
$= \boxed{1-12n+48n^2-64n^3}$

7.) $(5m^2-2n^4)^4$

$_4C_0\, (5m^2)^4(-2n^4)^0 = 1\cdot 625m^8\cdot 1$
$_4C_1\, (5m^2)^3(-2n^4)^1 = 4\cdot 125m^5\cdot -2n^4$
$_4C_2\, (5m^2)^2(-2n^4)^2 = 6\cdot 25m^4\cdot 4n^8$
$_4C_3\, (5m^2)^1(-2n^4)^3 = 4\cdot 5m^2\cdot -8n^{12}$
$_4C_4\, (5m^2)^0(-2n^4)^4 = 1\cdot 1\cdot 16n^{16}$
$= \boxed{625m^8-1000m^5n^4+600m^4n^8-160m^2n^{12}+16n^{16}}$

8.) $(4a^4+3b^3)^5$

$_5C_0\, (4a^4)^5(3b^3)^0 = 1\cdot 1024a^{20}\cdot 1$
$_5C_1\, (4a^4)^4(3b^3)^1 = 5\cdot 256a^{16}\cdot 3b^3$
$_5C_2\, (4a^4)^3(3b^3)^2 = 10\cdot 64a^{12}\cdot 9b^6$
$_5C_3\, (4a^4)^2(3b^3)^3 = 10\cdot 16a^8\cdot 27b^9$
$_5C_4\, (4a^4)^1(3b^3)^4 = 5\cdot 4a^4\cdot 81b^{12}$
$_5C_5\, (4a^4)^0(3b^3)^5 = 1\cdot 1\cdot 243b^{15}$
$= \boxed{1024a^{20}+3840a^{16}b^3+5760a^{12}b^6+4320a^8b^9+1620a^4b^{12}+243b^{15}}$

II. Determine the specific term using the expanded binomial. Must show work!

9.) Third term of $(1+2y)^5$
$_5C_2\, (1)^3(2y)^2 = 10\cdot 1\cdot 4y^2$
$= \boxed{40y^2}$

10.) Second term of $(2x+y)^4$
$_4C_1\, (2x)^3(y)^1 = 4\cdot 8x^3\cdot y$
$= \boxed{32x^3y}$

11.) Fifth term of $(3x+4)^6$
$_6C_4\, (3x)^2(4)^4 = 15\cdot 9x^2\cdot 256$
$= \boxed{34560x^2}$

12.) Middle term of $(x-3)^8$ $\frac{8}{2}=4$
$_8C_4\, (x)^4(-3)^4 = 70x^4\cdot 81$
$= \boxed{5670x^4}$

13.) Fourth term of $(2-3a)^7$
$_7C_3\, (2)^4(-3a)^3 = 35\cdot 16\cdot -27a^3$
$= \boxed{-15120a^3}$

14.) Third term of $(4m+1)^4$
$_4C_2\, (4m)^2(1)^2 = 6\cdot 16m^2\cdot 1$
$= \boxed{96m^2}$

15.) Sixth term of $(x+3y)^5$
$_5C_5\, (x)^0(3y)^5 = 1\cdot 1\cdot 243y^5$
$= \boxed{243y^5}$

16.) Middle term of $(4x+5)^6$ $\frac{6}{2}=3$
$_6C_3\, (4x)^3(5)^3 = 20\cdot 64x^3\cdot 125$
$= \boxed{160{,}000x^3}$

17.) Fifth term of $(3m+n)^5$
$_5C_4\, (3m)^1(n)^4 = 5\cdot 3m\cdot n^4$
$= \boxed{15mn^4}$

18.) Fourth term of $(x-2y)^6$
$_6C_3\, (x)^3(-2y)^3 = 20x^3\cdot -8y^3$
$= \boxed{-160x^3y^3}$

19.) Third term of $(2x-5)^4$
$_4C_2\, (2x)^2(-5)^2 = 6\cdot 4x^2\cdot 25$
$= \boxed{600x^2}$

20.) Second term of $(x^2+4y)^5$
$_5C_1\, (x^2)^4(4y)^1 = 5\cdot x^8\cdot 4y$
$= \boxed{20x^8y}$