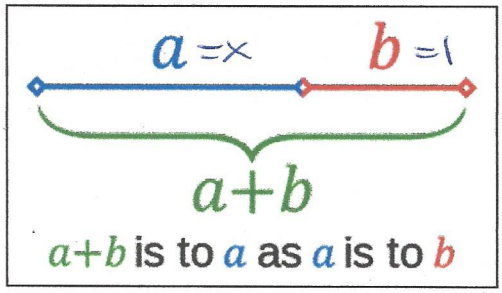
1.7 – Recursive and Special Sequences

Specific Sequence # 3 – Fibonacci Sequence

The following terms are in the Fibonacci Sequence → 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , __?__ …

- What is the pattern to this sequence? the sum of the two previous terms
- What is the missing term (value of the ?) to this sequence? $55+89=144$ → $? = 144$
- The Golden Ratio is a phenomena that is associated with the Fibonacci Sequence →



The Golden Ratio actually equals a specific number, let's figure it out:

$\frac{1+x}{x} = \frac{x}{1}$ (cross multiply) → $1+x = x^2$

$-1-x \quad -1-x$

$x^2 - x - 1 = 0$

$a=1,\ b=-1,\ c=-1$ } Quad. Formula

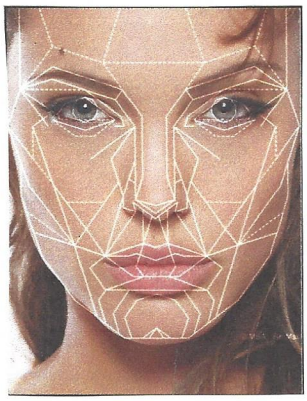
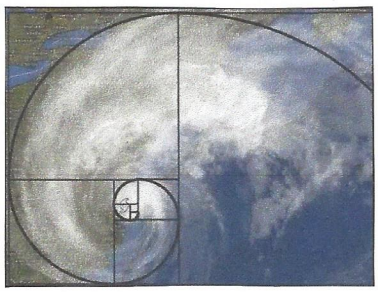
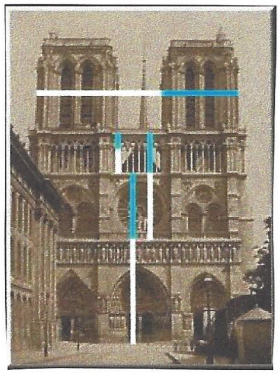
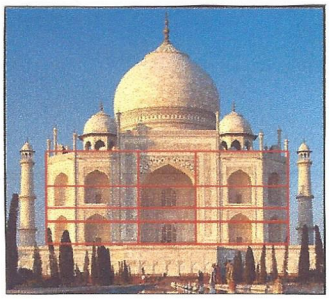
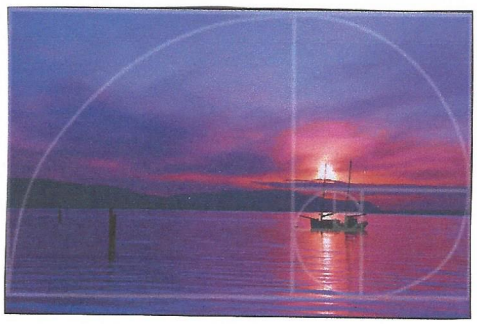
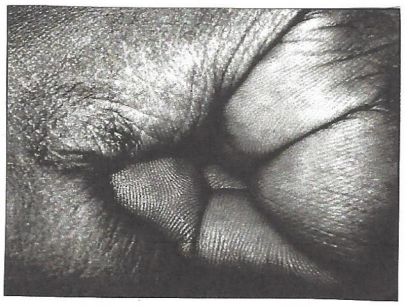
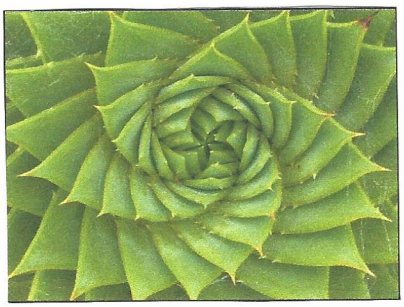
$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{1 \pm \sqrt{(-1)^2-4(1)(-1)}}{2(1)}$

$\frac{1+\sqrt{5}}{2} \approx 1.61803398$ ↓ φ (phi)

Example 1: Create a collage of at least 10 pictures demonstrating Fibonacci Sequence/Golden Ratio that occur naturally in the human world. Refer to your "Cut-out Sheet" to create your collage.



Specific Sequence # 4 – Recursive Sequence (Formula)

– **recursive sequence** → a sequence that uses a previous term to get to next term where sequence doesn't necessarily have to be consistently adding/subtracting (like arithmetic sequences) or consistently multiplying/dividing (like geometric sequences)

Note: The following are examples of recursive sequences → $a_n = 2a_{n-1} + 5$ or $a_{n+1} = 2a_n + 5$

Example 2: Find the first five terms of the given sequence. Write your answer as a sequence.

a.) $a_{n+1} = 3a_n - 2$; $a_1 = 4$

$a_1 = 4$

$a_2 = 3(4) - 2 = 10$

$a_3 = 3(10) - 2 = 28$

$a_4 = 3(28) - 2 = 82$

$a_5 = 3(82) - 2 = 244$

↓

4, 10, 28, 82, 244, ...

b.) $a_n = -2(a_{n-1} + 6)$; $a_1 = 3$

$a_1 = 3$

$a_2 = -2(3+6) = -18$

$a_3 = -2(-18+6) = 24$

$a_4 = -2(24+6) = -60$

$a_5 = -2(-60+6) = 108$

↓

3, −18, 24, −60, 108, ...

c.) $a_n = \frac{(a_{n-1})^2}{2}$; $a_1 = -4$

$a_1 = -4$

$a_2 = \frac{(-4)^2}{2} = 8$

$a_3 = \frac{(8)^2}{2} = 32$

$a_4 = \frac{(32)^2}{2} = 512$

$a_5 = \frac{(512)^2}{2} = 131072$

↓

−4, 8, 32, 512, 131072, ...

d.) $a_{n+1} = 4a_n + 2n$; $a_1 = 5$

$a_1 = 5$

$a_2 = 4(5) + 2(1) = 22$

$a_3 = 4(22) + 2(2) = 92$

$a_4 = 4(92) + 2(3) = 374$

$a_5 = 4(374) + 2(4) = 1504$

↓

5, 22, 92, 374, 1504, ...

Example 3: Complete the word problem below using recursive sequences.

Caleb discovered that there were 225 dandelions in his garden on the first Saturday of spring. He had time to pull out 100, but by the next Saturday, there were twice as many as he had left. Each Saturday in spring, he removed 100 dandelions, only to find that the number of remaining dandelions had doubled by the following Saturday.

a.) Write a recursive formula for the number of dandelions Caleb finds in his garden each Saturday.

d_n = # of dandelions at begin of nth Sat

Caleb will pull at 100 → $d_n - 100$

d_{n+1} = # of dandelions of next Sat but will be twice as much

↳ $d_{n+1} = 2(d_n - 100)$ → $d_{n+1} = 2d_n - 200$

b.) Find the number of dandelions Caleb would find on the fourth Saturday. $d_4 = ?$

$d_1 = 225$

$d_2 = 2(225) - 200 = 250$

$d_3 = 2(250) - 200 = 300$

$d_4 = 2(300) - 200 = 400$

↳ 400 dandelions remain on 4th Sat

– **explicit sequence** → a sequence that uses the value of a specified term based on its position

Example 4: Complete each problem involving explicit and recursive sequences.

a.) Write the explicit and recursive formula:

i.) 4, 10, 16, 22, 28, ... $a_1 = 4$ add by 6

explicit → $a_n = 4 + 6(n-1)$

recursive → $a_n = a_{n-1} + 6$; $a_1 = 4$

ii.) 3, 6, 12, 24, 48, ... $a_1 = 3$ mult by 2

explicit → $a_n = 3(2)^{n-1}$

recursive → $a_n = 2a_{n-1}$; $a_1 = 3$

b.) Which formula has the biggest fourth term and by how much?

explicit $a_n = 2n^2 - 6n + 3$ → $a_4 = 2(4)^2 - 6(4) + 3 = 11$

recursive $a_n = 4a_{n-1} - 5$; $a_1 = 2$

$a_2 = 4(2) - 5 = 3$

$a_3 = 4(3) - 5 = 7$

$a_4 = 4(7) - 5 = 23$

⇒ recursive formula's fourth term biggest by 12