

Adv. Functions - 1.7 WS: Recursive Sequences

1) $a_{n+1} = 6 - a_n$, $a_1 = -4$

$$a_1 = -4$$

$$a_2 = 6 - (-4) = 10$$

$$a_3 = 6 - 10 = -4$$

$$a_4 = 6 - (-4) = 10$$

$$a_5 = 6 - 10 = -4$$

↓

$$\boxed{-4, 10, -4, 10, -4, \dots}$$

2) $a_{n+1} = 2a_n + 7$, $a_1 = -3$

$$a_1 = -3$$

$$a_2 = 2(-3) + 7 = 1$$

$$a_3 = 2(1) + 7 = 9$$

$$a_4 = 2(9) + 7 = 25$$

$$a_5 = 2(25) + 7 = 57$$

↓

$$\boxed{-3, 1, 9, 25, 57, \dots}$$

3) $a_{n+1} = a_n - 3n + 1$, $a_1 = 5$

$$a_1 = 5$$

$$a_2 = 5 - 3(1) + 1 = 3$$

$$a_3 = 3 - 3(2) + 1 = -2$$

$$a_4 = -2 - 3(3) + 1 = -10$$

$$a_5 = -10 - 3(4) + 1 = -21$$

↓

$$\boxed{5, 3, -2, -10, -21, \dots}$$

4) $a_n = 4a_{n-1} - 2$, $a_1 = 2$

$$a_1 = 2$$

$$a_2 = 4(2) - 2 = 6$$

$$a_3 = 4(6) - 2 = 22$$

$$a_4 = 4(22) - 2 = 86$$

$$a_5 = 4(86) - 2 = 342$$

↓

$$\boxed{2, 6, 22, 86, 342, \dots}$$

5) $a_n = 3(a_{n-1} - 2)$, $a_1 = 7$

$$a_1 = 7$$

$$a_2 = 3(7 - 2) = 15$$

$$a_3 = 3(15 - 2) = 39$$

$$a_4 = 3(39 - 2) = 111$$

$$a_5 = 3(111 - 2) = 327$$

↓

$$\boxed{7, 15, 39, 111, 327, \dots}$$

6) $a_n = 4 - (a_{n-1} + 3)$, $a_1 = 2$

$$a_1 = 2$$

$$a_2 = 4 - (2 + 3) = 3$$

$$a_3 = 4 - (3 + 3) = -2$$

$$a_4 = 4 - (-2 + 3) = 3$$

$$a_5 = 4 - (3 + 3) = -2$$

↓

$$\boxed{-2, 3, -2, 3, -2, \dots}$$

$$7.) a_{n+1} = a_n - a_{n-1}, a_1 = -1, a_2 = -1$$

$$a_1 = -1$$

$$a_2 = -1$$

$$a_{3+1} = a_4 = a_3 - a_{3-1}$$

$$a_4 = a_3 - a_2$$

$$a_4 = 0 - (-1) = 1$$

$$\rightarrow \boxed{-1, -1, 0, 1, 1, \dots}$$

$$a_{2+1} = a_3 = a_2 - a_{2-1}$$

$$a_3 = a_2 - a_1$$

$$a_3 = -1 - (-1) = 0$$

$$a_{4+1} = a_5 = a_4 - a_{4-1}$$

$$a_5 = a_4 - a_3$$

$$a_5 = 1 - 0 = 1$$

$$8.) a_{n+1} = -4a_n + a_{n-1}, a_1 = 3, a_2 = -5$$

$$a_1 = 3$$

$$a_2 = -5$$

$$a_4 = -4a_3 + a_2$$

$$a_4 = -4(23) + (-5) = -97$$

$$a_3 = -4a_2 + a_1$$

$$a_3 = -4(-5) + 3 = 23$$

$$a_5 = -4a_4 + a_3$$

$$a_5 = -4(-97) + 23 = 411$$

$$\rightarrow \boxed{3, -5, 23, -97, 411, \dots}$$

$$9.) a_n = 2a_{n-1} + 3a_{n-2}, a_1 = 1, a_2 = 2$$

$$a_1 = 1$$

$$a_2 = 2$$

$$a_4 = 2a_3 + 3a_2$$

$$a_4 = 2(7) + 3(2) = 20$$

$$\rightarrow \boxed{1, 2, 7, 20, 50, \dots}$$

$$a_3 = 2a_2 + 3a_1$$

$$a_3 = 2(2) + 3(1) = 7$$

$$a_5 = 2a_4 + 3a_3$$

$$a_5 = 2(20) + 3(7) = 50$$

$$10.) 1, 3, 6, 10, 15, \dots$$

$$\hookrightarrow \text{recursive formula: } a_n = a_{n-1} + n, a_1 = 1$$

$$\text{check: } a_1 = 1$$

$$a_2 = 1 + 2 = 3 \checkmark$$

$$a_3 = 3 + 3 = 6 \checkmark$$

$$a_4 = 6 + 4 = 10 \checkmark$$

$$a_5 = 10 + 5 = 15 \checkmark$$

$$11.) f(x) = 2x + 3, x_0 = 1 \rightarrow$$

$$x_1 = f(x_0) = f(1) = 2(1) + 3 = 5$$

$$x_2 = f(x_1) = f(5) = 2(5) + 3 = 13$$

$$x_3 = f(x_2) = f(13) = 2(13) + 3 = 29$$

$$\rightarrow \boxed{5, 13, 29}$$

$$12) a_n = 2(3a_{n-1} + 5) - 10, a_1 = 4$$

$$a_n = 6a_{n-1} + 10 - 10$$

$$a_n = 6a_{n-1} \longrightarrow a_1 = 4$$

$$a_2 = 6(4) = 24 \longrightarrow$$

$$a_3 = 6(24) = 144$$

explicit formula:
 $a_n = 4(6)^{n-1}$

$$13) a_{n+1} = 5a_n + 3n - 1, a_1 = -2 \rightarrow S_5 = ?$$

$$\textcircled{1} a_1 = -2$$

$$a_2 = 5(-2) + 3(1) - 1 = -8$$

$$a_3 = 5(-8) + 3(2) - 1 = -35$$

$$a_4 = 5(-35) + 3(3) - 1 = -167$$

$$a_5 = 5(-167) + 3(4) - 1 = -490$$

$$\textcircled{2} S_5 = -2 + (-8) + (-35) + (-167) + (-490)$$

$S_5 = -702$

$$14) a_n = (1.006 \cdot a_{n-1}) - 678.79, a_1 = 100,000$$

$$a_1 = 100,000$$

$$a_2 = (1.006 \cdot 100,000) - 678.79 = \$99,921.21$$

$$a_3 = (1.006 \cdot 99,921.21) - 678.79 = \$99,841.95$$

$$a_4 = (1.006 \cdot 99,841.95) - 678.79 = \$99,762.21$$

$$a_5 = (1.006 \cdot 99,762.21) - 678.79 = \boxed{\$99,681.99}$$

$$15) b_n = 1.05b_{n-1} - 10, b_1 = 1000$$

$$b_1 = 100$$

$$b_2 = 1.05(1000) - 10 = \$1040$$

$$b_3 = 1.05(1040) - 10 = \$1082$$

$$b_4 = 1.05(1082) - 10 = \$1126.10$$

$$b_5 = 1.05(1126.1) - 10 = \$1172.41$$

$$b_6 = 1.05(1172.41) - 10 = \$1221.03$$

$$b_7 = 1.05(1221.03) - 10 = \boxed{\$1272.08}$$

$$16) C(x) = 1.06x, x_0 = 2000$$

$$x_1 = f(x_0) = 1.06(2000) = 2120$$

$$x_2 = f(x_1) = 1.06(2120) = 2247.2$$

$$x_3 = f(x_2) = 1.06(2247.2) = 2382.03$$

$$x_4 = f(x_3) = 1.06(2382.03) = 2524.95$$

$$x_5 = f(x_4) = 1.06(2524.95)$$

$$= \boxed{\$2676.45}$$