1.6 – Geometric Sequences and Series (F and I) Word Problems

Use the following formulas for the word problems below:

$$a_n = a_1(r)^{n-1} \qquad S_n = \frac{a_1(1-r^n)}{(1-r)} \qquad S = \frac{a_1}{(1-r)}$$

I. In the following set of examples, assume all geometric series are finite.

1.) A one-ton ice sculpture is melting at a rate of 30% of its original weight per hour. How much of the sculpture, in pounds, will be left when standing inside for 6 hours?

$a_1 = 1$ ton $= 2000$ lbs

$r = 1 - .3 = .7$ (70% of weight)

$a_6 = 2000(.7)^5$

$= \boxed{336.1 \text{ lbs}}$

2.) A ball is dropped at specific height, in feet, and then rebounds. On the eighth bounce, the ball's height is 0.02 feet. On the fifth bounce, the ball's height is 0.92 feet. What percent is the ball rebounding from its original height?

$a_8 = .02$, $a_5 = .92$ } gap → system! $r = ?$

$\div \begin{cases} .02 = a_1 r^7 \\ .92 = a_1 r^4 \end{cases}$

$\sqrt[3]{.0217} = \sqrt[3]{r^3}$ → $r = .28$

$= \boxed{28\%}$

3.) Benny got a new job that guarantees him a raise every year. His annual pay raise is always 4% of his salary at that time. After four years at the job, Benny's salary is \$53,814. What was Benny's starting salary when he was hired?

$a_5 = 53814$ $r = 1.04$

$a_1 = ?$

(doesn't get raise until after 1st year)

$\frac{53814}{(1.04)^4} = \frac{a_1(1.04)^4}{(1.04)^4}$

$=$ $\boxed{\text{about \$46000 for first year}}$

4.) A culture of bacteria initially has 6,000 bacteria and its sizes increases by 18% every hour. How many hours will it take for 19,112.8434 bacteria to be in the culture?

$a_1 = 6000$ $r = 1 + .18 = 1.18$

$n = ?$

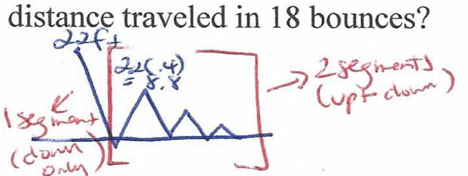
$\frac{19112.8434}{6000} = \frac{6000(1.18)^{n-1}}{6000}$

$3.1854739 = (1.18)^{n-1}$

$1 + \frac{\log 3.1854739}{\log 1.18} = \frac{(n-1)\log 1.18}{\log 1.18} + 1$

$n = 7.999\ldots \rightarrow \boxed{n = 8 \text{ hrs}}$

5.) A ball is dropped from a height of 22 feet. Each time it drops, it rebounds 40% of the height from which it is falling. What is the total distance traveled in 18 bounces?



$S_{18} = 22 + 2\left[\frac{8.8(1-.4^{18})}{(1-.4)}\right]$

$= 22 + 2(14.7)$

$= \boxed{51.4 \text{ ft}}$

6.) Heavy rain caused a river to rise. The river rose three inches the first day, then it rose six inches the second day, and so on. How many days did it take for the river to rise 765 inches?

$a_1 = 3$, $a_2 = 6$ → $n = ?$

$r = 2$

$S_n = 765$

$765 = \frac{3(1-2^n)}{(1-2)}$

$765 = -3(1-2^n)$

$-255 = 1 - 2^n$

$-256 = -2^n$

$256 = 2^n$

$\frac{\log 256}{\log 2} = \frac{n \log 2}{\log 2}$

$n = 8$ → $\boxed{8 \text{ days}}$

II. In the following set of examples, assume all geometric series are infinite.

7.) A child on a swing is given a big push. She travels 12 feet on the first swing but only $\frac{5}{6}$ as far on each successive swing. How far (total distance) does she travel before the swing stops?

$a_1 = 12$ $r = \frac{5}{6}$ → $S = ?$

$S = \frac{12}{(1-(\frac{5}{6}))} = \boxed{72 \text{ ft}}$

8.) A ball is thrown 12 meters in the air. The ball rebounds a percent of the distance it falls. If the ball's total vertical distance is 480 meters, what percent is its rebound?

$a_1 = 24$ (goes up then down)

$S = 480$ → $r = ?$

$480 = \frac{24}{1-r}$

$480 - 480r = 24$

$\frac{-480r}{-480} = \frac{-456}{-480}$

$r = .95$

$= \boxed{95\%}$

9.) Jack is working on a math problem on his homework and needs help. This is the problem he needs help on: $0.\overline{4} + 2.\overline{351}$.
What answer (as a fraction) should Jack put on his paper?

$\begin{array}{r} .444 \\ +2.351 \\ \hline 2.795 \end{array}$

$2.7 + [.095 + .00095 + \ldots]$

$a_1 = .095$

$r = \frac{.00095}{.095} = .01$

$S = 2.7 + \left[\frac{.095}{1-.01}\right]$

$= \boxed{\frac{1384}{495}}$