Adv. Functions – 1.6 WS: Geometric WP's

1) $a_1 = 5$, $a_2 = 5.50$, $a_3 = 6.05$ $\Rightarrow r = \frac{5.50}{5} = 1.1$

$n = 52$ (wks in a year)

$a_{52} = ?$

$a_{52} = 5(1.1)^{51}$

$= \boxed{\$645.65}$

2) $a_1 = 5$ $r = 5$ $n = 7$ (days in wk)

$S_7 = ?$

$S_7 = \frac{5(1-5^7)}{(1-5)}$

$= \boxed{97,655 \text{ people}}$

3) $a_1 = 200$

$r = 2$ (doubles)

$a_7 = ?$

$n=1$

$n=2 \rightarrow 2$ hrs

$n=3 \rightarrow 4$ hr

$n=4 \rightarrow 6$ hr

$n=5 \rightarrow 8$ hr

$n=6 \rightarrow 10$ hrs

$n=7 \rightarrow 12$ hrs

$a_7 = 200(2)^6$

$= \boxed{12,800 \text{ bacterium}}$

4) $a_4 = 10.62$ (for 1983)

$a_8 = 11.92$ (for 1987)

$a_{21} = ?$ (for 2000)

① $\begin{cases} 11.92 = a_1 r^7 \\ 10.62 = a_1 r^3 \end{cases} \div$

$1.122 = r^4$

$\sqrt[4]{1.122} = \sqrt[4]{r^4}$

$r = 1.029$

② $10.62 = a_1(1.029)^3$

$a_1 = 9.746$

③ $a_{21} = 9.746(1.029)^{20}$

$= \boxed{17.3 \text{ million ppl}}$

5) $a_0 = 40,000$

$a_1 = 41,600$ $\Rightarrow r = \frac{41600}{40,000}$

$r = 1.04$

$a_5 = ?$ (4 more raises)

$a_5 = 41600(1.04)^4$

$= \boxed{\$48,666.12}$

6) $a_1 = 8$

$S_n = 15.75$

$r = .5$

$n = ?$

$15.75 = \frac{8(1-.5^n)}{(1-.5)}$

$15.75 = \frac{8(1-.5^n)}{.5}$

$7.875 = 8(1-.5^n)$

$.984375 = 1-.5^n$

$-.015625 = -.5^n$

$\frac{\log .015625}{\log .5} = \frac{n \log .5}{\log .5}$

$n = 6$

$\rightarrow \boxed{6 \text{ swings}}$

7) $a_1 = 27$ $\quad \frac{8}{27} = \frac{27(r)^3}{27}$

$a_4 = 8$ $\quad \sqrt[3]{\frac{8}{27}} = \sqrt[3]{r^3}$

$r = ?$ $\quad \boxed{r = \frac{2}{3} \text{ or } 66.7\%}$

8) $r = 1.05$ (raise is after 1st year)

$S_{40} = 3624000$

$a_1 = ?$

$3624000 = \frac{a_1(1-1.05^{40})}{(1-1.05)}$

$\frac{3624000}{120.7997742} = \frac{a_1(120.7997742)}{120.7997742}$

$a_1 = 30,000.056$ or

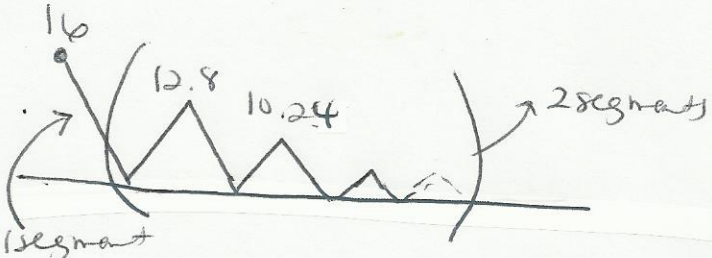
$\approx \boxed{\$30,000}$

9) $a_1 = 16(.8) = 12.8$

$r = .8$ (for 80%)

$n = 15$

$S_{15+16} = ?$



$\Rightarrow 16 + 2\left(\frac{12.8(1-.8^{15})}{(1-.8)}\right)$

$= \boxed{139.5 \text{ ft}}$

10) $a_1 = 8$ $\quad r = \frac{4}{5}$ $\quad S = ?$

$S = \frac{8}{(1-(\frac{4}{5}))}$

$= \boxed{40 \text{ ft}}$

11) $a_1 = 90$ $\quad S = 900$ $\quad r = ?$

$900 = \frac{90}{1-r}$

$900 - 900r = 90$

$-900 \quad\quad -900$

$-900r = -810$

$\frac{-900}{-900} \quad \frac{-810}{-900}$

$r = .9$

$\rightarrow \boxed{90\%}$

12) $a_1 = 24$ $\quad S = 72$ $\quad a_3 = ?$

① $72 = \frac{24}{1-r}$ $\quad$ ② $a_3 = 24(\frac{2}{3})^2$

$72 - 72r = 24$ $\quad = \boxed{10.7 \text{ cm}}$

$-72r = -48$

$\frac{-72}{-72} \quad \frac{-48}{-72}$

$r = \frac{2}{3}$

13) $.6\,7272\ldots$

$= .6 + [.072 + .00072 + \cdots]$

$a_1 = .072$ $\quad r = \frac{.00072}{.072} = .01$

$= .6 + \left(\frac{.072}{1-.01}\right) = \frac{37}{55}$

So $\boxed{\text{Susan is correct}}$

14) $S = 30$ $\quad r = .7$

$a_1 = ?$

$30 = \frac{a_1}{1-.7}$

$30 = \frac{a_1}{.3}$

$a_1 = 30(.3)$

$= \boxed{9 \text{ ft}}$

15) $a_1 = 2$

$r = \frac{2}{3}$ (what is left after removing $\frac{1}{3}$)

$S = ?$

$S = \frac{2}{(1-\frac{2}{3})}$

$= \boxed{6 \text{ cups of sugar}}$