1.5 – Geometric Series (Finite and Infinite)

Specific Series # 2 – Finite Geometric Series

– **geometric series** → the indicated Sum of terms in a geometric sequence

where it's represented by the following formula: $S_n = \dfrac{a_1\left(1 - r^n\right)}{(1 - r)}$

S_n: Sum of first n terms; a_1: first term; r: common ratio; r^n: common ratio raised to nth power.

* Reminder – Don't forget to put ( ) around any "r" that is a negative # or a fractional #

Example 1: Find the sum of each finite geometric series.

a.) $a_1 = 8,\ r = -3,\ n = 7$	b.) $a_1 = 1{,}280$, $a_9 = 5$	c.) $4 + 24 + \ldots + 31{,}104$	d.) $\sum_{n=4}^{18} 2(3)^{n-1}$
$S_7 = \dfrac{8(1-(-3)^7)}{(1-(-3))}$ $\boxed{S_7 = 4376}$	$r = ?$ ① $\dfrac{5}{1280} = \dfrac{1280(r)^8}{1280}$ $\sqrt[8]{\dfrac{1}{256}} = \sqrt[8]{r^8}$ $r = \frac{1}{2}$ ② $S_9 = \dfrac{1280(1-(\frac{1}{2})^9)}{(1-(\frac{1}{2}))}$ $\boxed{S_9 = 2555}$	a_1, $r = \frac{24}{4} = 6$, a_n ① $\dfrac{31104}{4} = \dfrac{4(6)^{n-1}}{4}$ $1 + \dfrac{\log 7776}{\log 6} = \dfrac{(n-1)\log 6}{\log 6} + 1$ $n = 6$ ② $S_6 = \dfrac{4(1-6^6)}{(1-6)}$ $\boxed{S_6 = 37324}$	r $a_1 = 2(3)^{4-1} = 54$ $n = 18-4+1 = 15$ $S_{15} = \dfrac{54(1-3^{15})}{(1-3)}$ $\boxed{S_{15} = 387420462}$

Example 2: Use the finite geometric series formula to complete each problem.

a.) The sum of first 8 terms is 39,360 and the common ratio is 3. What is the first term?	b.) The first term is – 3 and the common ratio is 4. How many terms were added together to get a sum of – 16,383?
$S_8 = 39360$, $n = 8$, $r = 3 \rightarrow a_1 = ?$ $39360 = \dfrac{a_1(1-3^8)}{(1-3)}$ $-2 \cdot 39360 = \dfrac{a_1(-6560)}{-2} \cdot -2$ $\dfrac{-78720}{-6560} = \dfrac{-6560\, a_1}{-6560}$ $\boxed{a_1 = 12}$	$a_1 = -3$, $r = 4$, $S_n = -16383 \rightarrow n = ?$ $-16383 = -3\dfrac{(1-4^n)}{(1-4)}$ $-16383 = \dfrac{\cancel{-3}(1-4^n)}{\cancel{-3}}$ $-16383 = 1 - 4^n$ $-16384 = -4^n$ $16384 = 4^n$ $\dfrac{\log 16384}{\log 4} = \dfrac{n \log 4}{\log 4}$ $n = 7$ $\boxed{7 \text{ terms}}$

Specific Series # 3 – Infinite Geometric Series

– **infinite geometric series** → the indicated partial sum of a geometric series

where it's represented by the following formula: $$S = \frac{a_1}{(1-r)}$$

▪ An infinite geometric series can do TWO THINGS:

1.) Converge → $-1 < r < 1$ (between) where sum = # ("merges")

2.) Diverge → not $-1 < r < 1$ where sum = DNE ("splits")

(DNE = does not exist)

Example 3: Determine if each series converges or diverges. If it's convergent, state the sum.

a.) $2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\dots$ a_1 $\quad r=\frac{\frac{1}{2}}{2}=\frac{1}{4}$ ✓ $S=\frac{2}{(1-\frac{1}{4})}=\frac{8}{3}$ → $\boxed{\text{converges to } \frac{8}{3}}$	b.) $\frac{1}{2}+\frac{3}{2}+\frac{9}{2}+\frac{27}{2}+\dots$ a_1 $\quad r=\frac{\frac{3}{2}}{\frac{1}{2}}=3$ X S = DNE → $\boxed{\text{diverges}}$	c.) $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$ $a_1=5$ $\quad r=\frac{1}{2}$ ✓ $S=\frac{5}{(1-\frac{1}{2})}=10$ → $\boxed{\text{converges to } 10}$	d.) $\sum_{n=1}^{\infty} -2\left(\frac{4}{3}\right)^{n-1}$ $a_1=-2$ $\quad r=\frac{4}{3}=1.\overline{3}$ X S = DNE → $\boxed{\text{diverges}}$

Example 4: Use the infinite geometric series formula to complete each problem.

a.) The sum of an infinite geometric series is 81, and its common ratio is ⅔. What is the first term? $S=81$, $r=\frac{2}{3} \rightarrow a_1=?$ $81=\frac{a_1}{(1-\frac{2}{3})}$ $81 \times \frac{a_1}{\frac{1}{3}}$ $\boxed{a_1=27}$	b.) The first term in an infinite geometric series is – 34, and its sum is – 42.5. What is common ratio? $a_1=-34$ $\quad S=-42.5 \rightarrow r=?$ $-42.5 \times \frac{-34}{1-r}$ $-42.5+42.5r=-34$ $+42.5 \qquad +42.5$ $\frac{42.5r}{42.5}=\frac{8.5}{42.5}$ $\boxed{r=.2}$	c.) Rewrite $1.\overline{42}$ as a series and then determine its fraction. 1.424242... $\rightarrow \underbrace{1}_{\text{not part of series (doesn't repeat)}}+[\underbrace{.42+.0042+\dots}_{\text{infinite geometric series (it repeats)}}]$ $a_1=.42$ $r=\frac{.0042}{.42}=.01$ $S=1+\left(\frac{.42}{1-.01}\right)$ $=\boxed{\frac{47}{33}}$