Advanced Functions – 1.5 WS: Geometric (F and I) Series Name: Key

I. Find sum for each finite geometric series described. Must show work!

1.) $a_1 = 2$, $r = 2$, $n = 6$

$S_6 = \frac{2(1-2^6)}{(1-2)}$

$\boxed{S_6 = 126}$

2.) 64 – 96 + 144 – … to 8 terms

a_1 $\quad n = 8$

$r = -\frac{96}{64} = -\frac{3}{2}$

$S_8 = \frac{64(1-(-\frac{3}{2})^8)}{(1-(-\frac{3}{2}))}$

$\boxed{S_8 = -630.5 \text{ or } \frac{-1261}{2}}$

3.) $a_7 = -192$ and $r = -2$

① $\frac{-192}{(-2)^6} = \frac{a_1(-2)^6}{(-2)^6}$

$a_1 = -3$

② $S_7 = \frac{-3(1-(-2)^7)}{(1-(-2))}$

$\boxed{S_7 = -129}$

4.) $a_1 = \frac{1}{3}$ and $a_{10} = 6,561$

① $3 \cdot 6561 = \frac{1}{3}(r)^9 \cdot 3$

$19683 = r^9$

$\sqrt[9]{19683} = \sqrt[9]{r^9}$

$r = 3$

② $S_{10} = \frac{\frac{1}{3}(1-3^{10})}{(1-3)}$

$\boxed{S_{10} = 9841.\overline{3} \text{ or } \frac{29524}{3}}$

5.) – 7 – 14 – 28 – … – 3,584

① $-3584 = -7(2)^{n-1}$

$512 = (2)^{n-1}$

$\frac{\log 512}{\log 2} = \frac{(n-1)\log 2}{\log 2}$ $\quad +1 \quad +1$

$n = 10$

② $S_{10} = \frac{-7(1-2^{10})}{(1-2)}$

$\boxed{S_{10} = -7161}$

6.) $a_3 = -36$, $a_6 = -972$, $n = 5$

① $\frac{-972 = a_1 r^5}{-36 = a_1 r^2}$

$27 = r^3$

$r = 3$

② $-36 = a_1(3)^2$

$-36 = a_1 \cdot 9$

$a_1 = -4$

③ $S_5 = \frac{-4(1-3^5)}{(1-3)}$

$\boxed{S_5 = -484}$

II. Use the geometric series formulas to complete each problem. Must show work!

7.) If the sum of the first 8 terms of a finite series is 1,530 and the common ratio is 2, then what is the first term?

$1530 = \frac{a_1(1-2^8)}{(1-2)}$

$-1530 = -255a_1$

$\boxed{a_1 = 6}$

8.) If the sum of a finite series is 1,062,880, the first term is 4, and the common ratio is 3, then how many terms were there?

$1062880 = \frac{4(1-3^n)}{(1-3)}$

$-531440 = 1 - 3^n$

$-531441 = -3^n$

$\frac{\log 531441}{\log 3} = \frac{n \log 3}{\log 3}$

$n = 12$

$\boxed{12 \text{ terms}}$

9.) If the sum of an infinite series is 40 and the common ratio is 0.6, then what is the first term of the series?

$40 = \frac{a_1}{1-.6}$

$40 = \frac{a_1}{.4}$

$\boxed{a_1 = 16}$

10.) If the sum of an infinite series is – 48 and the first term is – 12, then what is the common ratio?

$-48 = \frac{-12}{1-r}$

$-48 + 48r = -12$

$48r = 36$

$\boxed{r = .75 \text{ or } \frac{3}{4}}$

III. Determine if each series converges or diverges. If it's convergent, state the sum.

11.) $6 + 4 + \frac{8}{3} + \dots$

$r = \frac{4}{6} = \frac{2}{3}$ ✓

$S = \frac{6}{1-(\frac{2}{3})}$

$S = 18$

→ $\boxed{\text{converges to } 18}$

12.) $4 - 8 + 16 - \dots$

$r = \frac{-8}{4} = -2$ ✗

$S = \text{DNE}$

→ $\boxed{\text{diverges}}$

13.) $-98 - 73.5 - 55.125 - \dots$

$r = \frac{-73.5}{-98} = .75$ ✓

$S = \frac{-98}{(1-.75)} = -392$

→ $\boxed{\text{converges to } -392}$

14.) $\frac{1}{3} + \frac{5}{6} + \frac{25}{12} + \dots$

$r = \frac{5}{6} \div \frac{1}{3} = 2.5$ ✗

$S = \text{DNE}$

→ $\boxed{\text{diverges}}$

15.) $\frac{1}{2} - \frac{3}{8} + \frac{9}{32} - \dots$

$r = -\frac{3}{8} \div \frac{1}{2} = -.75$ ✓

$S = \frac{\frac{1}{2}}{(1-(-.75))} = \frac{2}{7}$

→ $\boxed{\text{converges to } \frac{2}{7}}$

16.) $3 + 1 + \frac{1}{3} + \dots$

$r = \frac{1}{3}$ ✓

$S = \frac{3}{(1-(1/3))} = \frac{9}{2}$

→ $\boxed{\text{converges to } \frac{9}{2}}$

IV. Find the sum (if it exists) using the appropriate method.

17.) $\sum_{n=3}^{10} 4(-3)^{n-1}$

$a_1 = 4(-3)^{3-1} = 36$

$n = 10 - 3 + 1 = 8$

$S_8 = \frac{36(1-(-3)^8)}{(1-(-3))}$

$\boxed{S_8 = -59040}$

18.) $\sum_{n=1}^{\infty} 4\left(\frac{1}{5}\right)^{n-1}$

$r = \frac{1}{5}$ ✓

$S = \frac{4}{(1-(\frac{1}{5}))}$

$\boxed{S = 5}$

19.) $\sum_{n=2}^{7} \frac{1}{4}(2)^{n-1}$

$a_1 = \frac{1}{4}(2)^{2-1} = \frac{1}{2}$

$n = 7 - 2 + 1 = 6$

$S_6 = \frac{\frac{1}{2}(1-2^6)}{(1-2)}$

$\boxed{S_6 = 31.5 \text{ or } \frac{63}{2}}$

20.) $\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{5}{4}\right)^{n-1}$

$r = \frac{5}{4}$ or 1.25 ✗

$\boxed{S = \text{DNE}}$