# 1.4 – Geometric Sequences

**Specific Sequence # 2 – Geometric Sequence**

– **geometric sequence** → a sequence where the ratio between any two consecutive terms is a constant, called r, the common ratio

**Example 1: Complete each problem.**

a.) Is the given sequence geometric?
3 , 9 , 27 , 81 , …
If so, what is the value of r?

$\frac{9}{3}$ $\frac{27}{9}$ $\frac{81}{27}$
$=3$ $=3$ $=3$

Yes, $r = 3$

b.) Is the given sequence geometric?
96 , – 24 , 6 , – 1.5, …
If so, what is the value of r?

$\frac{-24}{96}$ $\frac{6}{-24}$ $\frac{-1.5}{6}$
$= -.25 = -.25 = -.25$

Yes, $r = -\frac{1}{4}$ or $-.25$

c.) A geometric sequence has $a_1 = 4$ and $r = 6$. What is the fourth term of the sequence?

$a_1 = 4$
$a_2 = 4 \times 6 = 24$
$a_3 = 24 \times 6 = 144$
$a_4 = 144 \times 6 = 864$

$a_4 = 864$

**"Nth Term Formula" of Geometric Sequence: Used to find ANY term of a geometric sequence**

Consider a geometric sequence whose first term is $a_1$ and whose common ratio is $r$:

| | | |
|---|---|---|
| $a_1$ | → | 1st term ( $a_1$ ) |
| $a_1 \cdot r$ | → | 2nd term ( $a_2 =$ $a_1 \cdot r$ ) |
| $a_1 \cdot r \cdot r = a_1 \cdot r^2$ | → | 3rd term ( $a_3 =$ $a_1 \cdot r^2$ ) |
| $a_1 \cdot r^2 \cdot r = a_1 \cdot r^3$ | → | 4th term ( $a_4 =$ $a_1 \cdot r^3$ ) |
| $a_1 \cdot r^3 \cdot r = a_1 \cdot r^4$ | → | 5th term ( $a_5 =$ $a_1 \cdot r^4$ ) |

**(General) nth term Formula:** $a_n = a_1 (r)^{n-1}$ → some important notes about this formula…

- formula will always be an exponential equation
- do not multiply $a_1$ # and r # together to simplify the formula
- put ( ) around any "r" that's a negative # or fractional #

**Example 2: Find what is indicated for each ~~arithmetic~~ geometric sequence.**

a.) $a_1 = 2$ and $r = 4$, find the 8th term

$a_8 = 2(4)^{8-1}$
$a_8 = 2(4)^7$
$a_8 = 32768$

b.) Find $a_5$ for the sequence
$-1, \frac{1}{4}, -\frac{1}{16}, \ldots$

$r = \frac{1}{4} \div -1 = -\frac{1}{4}$
$a_5 = -1\left(-\frac{1}{4}\right)^{5-1}$
$a_5 = -1\left(-\frac{1}{4}\right)^4$
$a_5 = -\frac{1}{256}$

c.) Write the nth term formula (equation) for the sequence
3 , – 36 , 432 , …

$a_1$ $r = \frac{-36}{3} = -12$

$a_n = 3(-12)^{n-1}$

* note: can't simplify as $a_n = (-36)^{n-1}$

**Example 3: Considering all given sequences are geometric – Find what is asked.**

a.) The 7th term of the sequence is 62,500 and the common ratio is 5. What is the first term?

$a_7 = 62500 \quad r = 5 \rightarrow a_1 = ?$

$62500 = a_1(5)^6$

$\frac{62500}{15625} = \frac{a_1 \cdot 15625}{15625}$

$\boxed{a_1 = 4}$

b.) What is the common ratio for the sequence where the first term is 96 and the 6th term is 3?

$a_1 = 96, \; a_6 = 3 \rightarrow r = ?$

$\frac{3}{96} = \frac{96(r)^5}{96}$

$\sqrt[5]{\frac{1}{32}} = \sqrt[5]{r^5}$

$\boxed{r = \frac{1}{2}}$

c.) Which term is 78,732 in the sequence of 4 , 12 , 36 , 108 , …? $\frac{12}{4} = 3$

$a_n = 78732, \; a_1 = 4, \; r = 3 \rightarrow n = ?$

$\frac{78732}{4} = \frac{4(3)^{n-1}}{4}$

$19683 = (3)^{n-1}$

$\frac{\log 19683}{\log 3} = \frac{(n-1)\log 3}{\log 3} + 1$

$n = 10 \rightarrow \boxed{10^{th} \text{ term}}$

d.) The fifth term in the sequence is 768 and the ninth term is 196,608. What is the third term of the sequence?

$a_5 = 768$, $a_9 = 196608$ } gap! → system of eq. put higher term # on top!

① ÷ ; ② ③

$\begin{cases} 196608 = a_1 r^8 \\ 768 = a_1 r^4 \end{cases}$

$\frac{256}{256} = \frac{1 \cdot r^4}{}$

$\sqrt[4]{256} = \sqrt[4]{r^4}$

$r = 4$

② $196608 = a_1 \cdot 4^8$

$\frac{196608}{65536} = \frac{a_1 \cdot 65536}{65536}$

$a_1 = 3$

③ $a_3 = 3(4)^2$

$\boxed{a_3 = 48}$

– **geometric means** → represent the terms between any two nonsuccessive terms of a geometric sequence

Ex: Circle the 4 geometric means between 2 and 468: 2 , (6 , 18 , 54 , 162) , 468 , …

**Example 4: Complete each problem. Assume both sequences are geometric.**

a.) Find the three geometric means for

$-6 \quad \boxed{-18 \; , -54 \; , -162} \quad -486$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

① $\frac{-486}{-6} = \frac{-6(r)^4}{-6}$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$r = 3$

② $a_2 = -6 \times 3 = -18$

$a_3 = -18 \times 3 = -54$

$a_4 = -54 \times 3 = -162$

b.) Find the two geometric means between – 20 and 1280.

$-20 \quad \boxed{80 \; , -320} \quad 1280$

$a_1 \quad a_2 \quad a_3 \quad a_4$

① $\frac{1280}{-20} = \frac{-20(r)^3}{-20}$

$\sqrt[3]{-64} = \sqrt[3]{r^3}$

$r = -4$

② $a_2 = -20 \times -4 = 80$

$a_3 = 80 \times -4 = -320$