1.3 – Arithmetic Sequences and Series Word Problems

Use the following formulas for the word problems below:

$$a_n = a_1 + d\cdot(n-1) \qquad S_n = \frac{n}{2}(a_1 + a_n) \qquad S_n = \frac{n}{2}(2a_1 + d\cdot(n-1))$$

1.) According to the National Education Association, teachers in the US earned an average of $21,700 per year in 1984. This amount increased by approximately $1,472 yearly. Determine the amount a US teacher earned in the year 2005.

$a_1 = 21700$ $n = 2005-1984 = 21$
$d = 1472$ $a_{21} = ?$

$a_{21} = 21700 + 1472(21-1)$
$= \boxed{\$51,140}$

2.) A drive-in theater has spaces for 20 cars in the first parking row, 22 in the second, 24 in the third, and so on. If there are 34 rows in the theater, find the number of cars that can be parked.

$a_1 = 20$ $a_2 = 22$ $a_3 = 24$
$d = 2$ $n = 34$ $S_{34} = ?$

① $a_{34} = 20 + 2(34-1)$
$a_{34} = 86$

② $S_{34} = \frac{34}{2}(20+86)$
$= \boxed{1802 \text{ cars}}$

3.) The purchase value of an office computer is $12,500. The value of the computer after 6 years is $1,850. What was the computer's annual depreciation?

$a_1 = 12500$ $n = 6+1$ (factoring 1st yr) $n = 7$
$a_7 = 1850$ $d = ?$

$1850 = 12500 + d(7-1)$
$1850 = 12500 + 6d$
$\frac{-10650}{6} = \frac{6d}{6}$
$d = -1775 \rightarrow$ loses $1775/year

4.) An architect designs a small theater with 13 seats in the first row. If the theater is has a seating capacity of 272 and 8 rows, how many seats are in the last row?

$a_1 = 13$ $S_8 = 204$ $(n=8)$ $a_n = ?$

$272 = \frac{8}{2}(13 + a_n)$
$\frac{272}{4} = \frac{4(13+a_n)}{4}$
$68 = 13 + a_n$
$-13 \quad -13$
$a_n = 55 \rightarrow$ 55 seats in last row

5.) A display of shoe boxes has 45 boxes the fourth row and has 94 boxes in the eleventh row. How many boxes are in the first row?

$a_4 = 45$
$a_{11} = 94$ } gap → system of eq.
$a_1 = ?$

① $45 = a_1 + 3d$
② $-94 = a_1 + 10d$
$-49 = -7d$
$-7 \quad -7$
$d = 7$

② $45 = a_1 + 3(7)$
$45 = a_1 + 21$
$-21 \quad -21$
$a_1 = 24$

24 seats in 1st row

6.) A radio station wants to give away money every day in August for a total of $124,000. They want to increase each day's giveaway amount by $100 but still keep the same total amount. How much should they give away on the first day?

$S_{31} = 124000$ $d = 100$ $a_1 = ?$ $(n=31)$

$124000 = \frac{31}{2}(2a_1 + 100(31-1))$
$2 \cdot 124000 = \frac{31}{2}(2a_1 + 3000) \cdot 2$
$248000 = 62a_1 + 93000$
$155000 = 62a_1$
$\frac{155000}{62} = \frac{62a_1}{62} \rightarrow \boxed{a_1 = \$2500}$

7.) A theater has 18 seats in the third row and 54 seats in the twelfth row. The last row in the theater has 94 seats. How many seats are in the entire theater?

$a_3 = 18$
$a_{12} = 54$ } gap → system of eq
$a_n = 94$

① $d = ?$
② $a_1 = ?$
③ $n = ?$
④ $S_n = ?$

① $18 = a_1 + 2d$
$-54 = a_1 + 11d$
$\frac{-36}{-9} = \frac{-9d}{-9}$
$d = 4$

② $18 = a_1 + 2(4)$
$18 = a_1 + 8$
$a_1 = 10$

③ $94 = 10 + 4(n-1)$
$84 = 4n - 4$
$88 = 4n$
$n = 22$

④ $S_{22} = \frac{22}{2}(10+94)$
$= \boxed{1144 \text{ seats}}$

8.) Gary wants to save money for a trip that costs $252. He gets paid every week at his job and starts the first week of saving with $12. He increases this amount every week by $4. How many weeks will it take Gary to save up for his trip?

$S_n = 252$ $a_1 = 12$ $d = 4$ $n = ?$

$252 = \frac{n}{2}(2(12) + 4(n-1))$
$252 = \frac{n}{2}(24 + 4n - 4)$
$2 \cdot 252 = \frac{n}{2}(4n+20) \cdot 2$
$504 = 4n^2 + 20n$
$4n^2 + 20n - 504 = 0$
$n^2 + 5n - 126 = 0$

$(n-9)(n+14) = 0$
$n = 9 \quad n = -14$
9 weeks (can't have neg weeks)