1.2 – Arithmetic Series and Sigma Notation By Hand

Introduction to (General) Series

– **series** → the indicated sum of terms in a sequence

- distinguish the difference – sequence → 6 , 12 , 18 , … , 300 where it contains commas

  series → 6 + 12 + 18 + … + 300 where it contains "+" signs (no commas)

- series notation – $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

  S_n: sum of first n terms

  $a_1 + a_2 + a_3 + \ldots + a_n$: sum of 1st term thru nth term (last term)

Example 1: Find the sum for each given sequence.

a.) Given: $a_n = 2n+6$

Find: S_4

$a_1 = 2(1)+6 = 8$

$a_2 = 2(2)+6 = 10$

$a_3 = 2(3)+6 = 12$

$a_4 = 2(4)+6 = 14$

$S_4 = 8+10+12+14$

$S_4 = 44$

b.) Given: $a_n = 4(3n-2)$

Find: S_3

$a_1 = 4(3\cdot1-2) = 4$

$a_2 = 4(3\cdot2-2) = 16$

$a_3 = 4(3\cdot3-2) = 28$

$S_3 = 4+16+28$

$S_3 = 48$

Specific Series # 1 – Arithmetic Series

– **arithmetic series** → the indicated sum of terms in an arithmetic sequence

where it's represented by the following formula: $S_n = \frac{n}{2}(a_1 + a_n)$

S_n: sum of first n terms

n: # of terms

a_1: 1st term

a_n: nth term (last term): $a_n = a_1 + d(n-1)$

Example 2: Find the sum of each arithmetic series.

a.) $a_1 = 8$, $a_n = 146$, $n = 24$

$S_{24} = \frac{24}{2}(8+146)$

$S_{24} = 12(154)$

$S_{24} = 1848$

b.) $a_1 = 21$, $d = -3$, $n = 17$

① $a_{17} = 21-3(17-1)$

$a_{17} = -27$

② $S_{17} = \frac{17}{2}(21-27)$

$S_{17} = -51$

c.) $9 + 22 + 35 + \ldots + 776$ ($d = 13$)

① $776 = 9+13(n-1)$

$767 = 13n-13$

$780 = 13n$

$n = 60$

② $S_{60} = \frac{60}{2}(9+776)$

$S_{60} = 23550$

Example 3: Use the arithmetic series formula to complete each problem.

a.) How many terms were added to get a sum of 5,958 when the first term is 8 and the last term in the series is 323?

$5958 = \frac{n}{2}(8+323)$

$2(5958) = \frac{n}{2}(331)\cdot 2$

$\frac{11916}{331} = \frac{331n}{331}$

$n = 36 \rightarrow$ **36 terms**

b.) What is the second term of an arithmetic series if the first term is 9, the nth term is 105, and the sum is 741?

① $741 = \frac{n}{2}(9+105)$

$2(741) = \frac{n}{2}(114)\cdot 2$

$\frac{1482}{114} = \frac{114n}{114}$

$n = 13$

② $105 = 9 + d(13-1)$

$\frac{96}{12} = \frac{12d}{12}$

$d = 8$

③ $a_1 = 9$

$a_2 = 9 + 8$

$a_2 = 17$

Sigma Notation (By Hand Method)

– **sigma notation** → a less lengthy and more concise way to write out a series

- The following is a simple representation of Sigma Notation:

$$\sum_{n=1}^{4} 3n = 3(1)+3(2)+3(3)+3(4) = 3+6+9+12 \Rightarrow S_4 = 30$$

Example 4: Find the sum.

a.) $\sum_{n=3}^{5} 4n+3$

$4(3)+3 = 15$

$4(4)+3 = 19$

$4(5)+3 = 23$

$S_3 = 15+19+23$

$S_3 = 57$

b.) $\sum_{n=1}^{5} 2n-4$

$2(1)-4 = -2$

$2(2)-4 = 0$

$2(3)-4 = 2$

$2(4)-4 = 4$

$2(5)-4 = 6$

$S_5 = -2+0+2+4+6$

$S_5 = 10$

c.) $\sum_{n=5}^{26} (5-3n)$ → linear eq ⇒ arithmetic series

$a_1 = 5-3(5) = -10$

$a_n = 5-3(26) = -73$

$n = 26-5+1 = 22$

$S_{22} = \frac{22}{2}(-10-73)$

$S_{22} = -913$