1.1 – Arithmetic Sequences

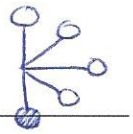
Introduction to (General) Sequences

– **sequence** → a list of a pattern in a certain order where each part of the pattern is called a term

• sequence notation – $a_1, a_2, a_3, \ldots, a_n$ (first term, second term, third term, nth term – function of the sequence)

* n is a "term #" so it's always a positive whole number.

• sequence representations – we can represent a sequence in three ways:

1.) picture → Ex:  , _____, _____ 

Rule: Add stem, color last one

2.) words → Ex: applying, bathing, cuddling, ducking, etching

Rule: verbs (ending) in alphabetical order

3.) numbers → Ex: 3, 10, 24, 45, 73, 108 (+7, +14, +21, +28, +35)

Rule: add by multiples of 7.

Specific Sequence # 1 – Arithmetic Sequence

– **arithmetic sequence** → a sequence where the difference between any two consecutive terms is a constant, called d, the common difference (keep add/subtract same #!)

Example 1: Complete each problem.

a.) Is the given sequence arithmetic? $-2, 4, 10, 16, \ldots$ If so, what is the value of d?	b.) Is the given sequence arithmetic? $18, 15, 12, 10, \ldots$ If so, what is the value of d?	c.) An arithmetic sequence has $a_1 = 20$ and $d = -4$. What is the fifth term of the sequence?
$4-(-2)$ $10-4$ $16-10$ $=6$ $=6$ $=6$	$15-18$ $12-15$ $10-2$ $=-3$ $=-3$ $=-2$	$a_1=20$ $a_2=20-4=16$ $a_3=16-4=12$ $a_4=12-4=8$ $a_5=8-4=4$
Yes, d = 6	NO	$a_5 = 4$

"Nth Term Formula" of Arithmetic Sequence: Used to find ANY term of an arithmetic sequence

Consider an arithmetic sequence whose first term is a_1 and whose common difference is d:

a_1	→	1st term (a_1)
$a_1 + d$	→	2nd term ($a_2 = a_1 + d$)
$a_1+d+d = a_1+2d$	→	3rd term ($a_3 = a_1 + 2d$)
$a_1+2d+d = a_1+3d$	→	4th term ($a_4 = a_1 + 3d$)
$a_1+3d+d = a_1+4d$	→	5th term ($a_5 = a_1 + 4d$)

(General) nth term Formula: $a_n = a_1 + (n-1)d$ or $a_n = a_1 + d(n-1)$ → some notes about this formula…

- formula will always be a linear equation
- make sure your final nth term formula is completely simplified (distribute + collect like terms)

Example 2: Find what is indicated for each arithmetic sequence.

a.) $a_1 = -6$ and $d = 7$, find the 16^{th} term ($n=16$)

$a_{16} = -6 + 7(16-1)$

$a_{16} = -6 + 7(15)$

$\boxed{a_{16} = 99}$

b.) Find a_{40} ($n=40$) for the sequence $-9, -17, -25, \ldots$

$a_1 = -9 \quad d = -17 - (-9) = -8$

$a_{40} = -9 - 8(40-1)$

$a_{40} = -9 - 8(39)$

$\boxed{a_{40} = 321}$

c.) Write the nth term formula (equation) for the sequence $8, 17, 26, 35, \ldots$

$a_1 = 8 \quad d = 17 - 8 = 9$

$a_n = 8 + 9(n-1)$

$a_n = 8 + 9n - 9$

$\boxed{a_n = 9n - 1}$

Example 3: Considering all given sequences are arithmetic – Find what is asked.

a.) The 20^{th} term of the sequence is 101 and the common difference is 3. What is the first term?

$a_{20} = 101, n = 20, d = 3 \rightarrow a_1 = ?$

$101 = a_1 + 3(20-1)$

$101 = a_1 + 57$

$-57 \quad -57$

$\boxed{a_1 = 44}$

b.) What is the common difference for the sequence where the first term is 13 and the 28^{th} term is – 149?

$a_1 = 13, a_{28} = -149, n = 28 \rightarrow d = ?$

$-149 = 13 + d(28-1)$

$-149 = 13 + 27d$

$-162 = 27d$

$\frac{-162}{27} = \frac{27d}{27}$

$\boxed{d = -6}$

c.) Which term of the sequence $1, 5, 9, \ldots.$ is 97?

$a_1 = 1, d = 5 - 1 = 4, a_n = 97 \rightarrow n = ?$

$97 = 1 + 4(n-1)$

$96 = 4n - 4$

$+4 \quad +4$

$\frac{100}{4} = \frac{4n}{4}$

$n = 25 \rightarrow \boxed{25^{th} \text{ term}}$

d.) The thirty-second term in the sequence is 534 and the fourteenth term is 228. What is the tenth term of the sequence?

① $d = ?$ ② $a_1 = ?$ ③ $a_{10} = ?$

$a_{32} = 534, a_{14} = 228 \rightarrow$

gap ⇒ setup system of eq!

① $\begin{cases} 534 = a_1 + 31d \\ 228 = a_1 + 13d \end{cases}$ −

$\frac{306}{18} = \frac{18d}{18}$

$d = 17$

② $534 = a_1 + 31(17)$

$534 = a_1 + 527$

$a_1 = 7$

③ $a_{10} = 7 + 17(10-1)$

$\boxed{a_{10} = 160}$

– **arithmetic means** → represent the terms b/w any two nonsuccessive terms of an arithmetic sequence

Ex: Circle the 3 arithmetic means between 30 and 74: 19, 30, (41, 52, 63), 74, 85, 96, …

Example 4: Complete each problem. Assume both sequences are arithmetic.

a.) Find the four arithmetic means for $16, \boxed{31, 46, 61, 76}, 91$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

① $91 = 16 + d(6-1)$

$91 = 16 + 5d$

$\frac{75}{5} = \frac{5d}{5}$

$d = 15$

② $a_2 = 16 + 15 = 31$

$a_3 = 31 + 15 = 46$

$a_4 = 46 + 15 = 61$

$a_5 = 61 + 15 = 76$

$a_6 = 76 + 15 = 91$ ✓

b.) Find the two arithmetic means between 52 and 10.

$52, \boxed{38, 24}, 10$

$a_1 \quad a_2 \quad a_3 \quad a_4$

① $10 = 52 + d(4-1)$

$10 = 52 + 3d$

$-42 = 3d$

$d = -14$

② $a_2 = 52 - 14 = 38$

$a_3 = 38 - 14 = 24$

$a_4 = 24 - 14 = 10$ ✓