Sequences and Series – (Infinite) Geometric Series

Specific Series #3 – (Infinite) Geometric Series

- **infinite geometric series** → the indicated Partial sum of a geometric series (doesn't stop/end)
  where it's represented by the following formula: $S = \frac{a_1}{(1 - r)}$

▪ (Formula) MUST meet TWO conditions:

1.) series must be an infinite geometric series (duh, obviously!)

** 2.) "r" must be b/w −1 and 1 but ≠ 0 **

Note – if the series does NOT meet the two conditions → write DNE (does not exist for answer)

Example 1: Find the sum (if it exists) of each infinite geometric series.

a.) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \ldots$ ($a_1 = 2$) → r = $\frac{2/5}{2} = .2$ then S = $\frac{2}{(1-.2)}$ (work) so S = $\frac{5}{2}$ or 2.5 (answer)

b.) 0.5 + 1.5 + 4.5 + 13.5 +.... → r = $\frac{1.5}{.5} = 3$ ($r > 1$) then S = X (work) so S = DNE (answer)

Example 2: Use the infinite geometric series formula to complete each problem.

a.) The sum of an infinite geometric series is 81, and its common ratio is ⅔. What is the first term?

$81 = \frac{a_1}{(1-2/3)}$

$81 = \frac{a_1}{1/3}$

$\frac{81}{3} = \frac{3a_1}{3}$

$a_1 = 27$

b.) The first term in an infinite geometric series is – 8 , and its sum is $-\frac{40}{3}$.

What is common ratio?

$-\frac{40}{3} = \frac{-8}{1-r}$ (cross multiply)

$-24 = -40(1-r)$

$-24 = -40 + 40r$

$+40 \quad +40$

$\frac{16}{40} = \frac{40r}{40}$

$r = 2/5$

c.) Find the sum for:

$\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$ $a_n = a_1 \cdot r^{n-1}$

a_1, r

$S = \frac{5}{(1-1/2)}$

$S = 10$

– Repeating decimals are considered infinite geometric series

Ex: $\frac{2}{3}$ = 0.666... or $0.\overline{6}$ → write it as a series = .6 + .06 + .006 + ... (at least 3 terms)

Example 3: Express each repeating decimal as a fraction (use formula – no typing in calculator…!)

a.) 0.888888... → .8 + .08 + .008 + ...

$a_1 = .8$

$r = \frac{.08}{.8} = .1$

$S = \frac{.8}{(1-.1)}$

$= \frac{8}{9}$

b.) $1.\overline{42}$ → 1 + [.42 + .0042 + .000042 + ...]

1 = not part of series; [.42 + .0042 + .000042 + ...] = infinite series

$a_1 = .42$

$r = \frac{.0042}{.42} = .01$

$S = 1 + \left(\frac{.42}{1-.01}\right)$

$= \frac{47}{33}$