Sequences and Series – (Finite) Geometric Series and Sigma Notation (W/ Calc)

Specific Series # 2 – (Finite) Geometric Series

- **geometric series** → the indicated <u>sum of terms in a geometric sequence</u>

where it's represented by the following formula: $$S_n = \frac{a_1(1 - r^n)}{(1 - r)}$$

S_n: <u>sum of first n terms</u> a_1: <u>first term</u> r^n: <u>common ratio raised to nth power</u>

* Reminder – Don't forget to put <u>( )</u> around any "r" that is a <u>negative #</u> or a <u>fractional #</u>

Example 1: Find S_n for each geometric series described.

a.) $a_1 = 8$, $r = -3$, $n = 7$ $S_7 = \frac{8(1-(-3)^7)}{(1-(-3))}$ $\boxed{S_7 = 4376}$	b.) $a_1 = -6$, $a_5 = 96$, $r = 2$ → ex info → n=5 $S_5 = \frac{-6(1-2^5)}{(1-2)}$ $\boxed{S_5 = -186}$	c.) 81 + 27 + 9 + … to 10 terms a_1 $r = \frac{27}{81} = \frac{1}{3}$ $n = 10$ $S_{10} = \frac{81(1-(\frac{1}{3})^{10})}{(1-(\frac{1}{3}))}$ $\boxed{S_{10} = 121.498 \text{ or } \frac{29524}{243}}$
d.) $a_8 = -458{,}752$ and $r = -4$ $a_1 = ?$ $-458752 = a_1(-4)^{8-1}$ $\frac{-458752}{-16384} = \frac{a_1 \cdot \cancel{-16384}}{\cancel{-16384}}$ $a_1 = 28$ $S_8 = \frac{28(1-(-4)^8)}{(1-(-4))}$ $\boxed{S_8 = -366996}$	e.) $a_1 = 1{,}280$ and $a_9 = 5$ n=9 r=? $\frac{5}{1280} = \frac{1280(r)^8}{1280}$ $\frac{1}{256} = r^8$ $\sqrt[8]{\frac{1}{256}} = \sqrt[8]{r^8} \rightarrow r = \frac{1}{2}$ $S_9 = \frac{1280(1-(\frac{1}{2})^9)}{(1-(1/2))}$ $\boxed{S_9 = 2555}$	f.) 4 + 24 + 144 + … + 31,104 a_1 $r = \frac{24}{4} = 6$ a_n $n = ?$ $\frac{31104}{4} = \frac{4(6)^{n-1}}{4}$ $7776 = 6^{n-1}$ $1 + \frac{\log 7776}{\log 6} = \frac{(n-1)\cancel{\log 6}}{\cancel{\log 6}} + 1$ $\rightarrow n = 6$ $S_6 = \frac{4(1-6^6)}{(1-6)}$ $\boxed{S_6 = 37324}$

Example 2: Find the indicated part of a (finite) geometric series given the series sum and other info.

Given: $S_8 = 39{,}360$ and $r = 3$ Find: a_1 $39360 = \frac{a_1(1-3^8)}{(1-3)}$ $-2 \cdot 39360 = \frac{a_1 \cdot -6560}{-2} \cdot -2$ $\frac{-78720}{-6560} = \frac{a_1 \cdot -6560}{-6560}$ $\boxed{a_1 = 12}$	Given: $S_n = -16{,}383$, $r = 4$, $a_1 = -3$ Find: n $-16383 = \frac{-3(1-4^n)}{(1-4)}$ $-16383 = \frac{\cancel{-3}(1-4^n)}{\cancel{-3}}$ $-16383 = 1 - 4^n$ $-1 \quad -1$ $\frac{-16384}{-1} = \frac{-4^n}{-1}$ $16384 = 4^n$ $\frac{\log 16384}{\log 4} = \frac{n \log 4}{\log 4}$ $\boxed{n = 7}$

Sigma Notation (By Calculator Method)

<table>
<tr><th>TI-8x Calculator (old operating system)</th><th>TI-8x Calculator (new operating system)</th></tr>
<tr><td>1.) 2nd STAT (LIST) – highlight MATH and press # 5 which should say sum (
2.) 2nd STAT (LIST) – highlight OPS and press # 5 which should say seq (
3.) Then type in the following:
a.) sequence written in terms of "x" ,
b.) x , lower limit # , upper limit #)
c.) ENTER 1 time
4.) number shown on the screen is the "answer"
5.) write your FINAL ANSWER as…
S_n = final answer , fill in "n" and "answer"
* To figure out "n" = upper – lower + 1</td><td>1.) 2nd STAT (LIST) – highlight MATH and press # 5 which should say sum (
2.) 2nd STAT (LIST) – highlight OPS and press # 5 which should say seq (
3.) Then fill out the following information:
a.) Variable: x
b.) Start: lower limit # ; End: upper limit #
c.) ENTER 4 times
4.) number shown on the screen is the "answer"
5.) write your FINAL ANSWER as…
S_n = final answer , fill in "n" and "answer"
* To figure out "n" = upper – lower + 1</td></tr>
<tr><td colspan="2">No matter which "system" (old or new) must show the calculator screen to represent your "work"!

Example: $\sum_{k=4}^{23} 4k^2 - 5$ → Calc Steps/Screen: **sum (seq ($4x^2 - 5$, x, 4, 23)** → $\boxed{S_{20} = 17{,}140}$</td></tr>
</table>

Example 3: Find the sum of the given geometric finite series using the three methods learned
All 3 methods should be the SAME EXACT ANSWER.
Make sure to write your answer as S_n = sum where you fill in "n" and the series' sum.

Given geometric finite series → $\sum_{n=4}^{7} 2(3)^{n-1}$

<table>
<tr><th>Method # 1 –
Sigma Notation By Hand</th><th>Method # 2 –
Geometric Finite Series Formula</th><th>Method # 3 –
Sigma Notation By Calculator</th></tr>
<tr><td>$= [2(3)^{4-1}] + [2(3)^{5-1}]$
$+ [2(3)^{6-1}] + [2(3)^{7-1}]$
$= \underbrace{54 + 162 + 486 + 1458}_{\text{4 terms to add up so } n=4}$
$\Rightarrow \boxed{S_4 = 2160}$</td><td>$S_n = \dfrac{a_1(1-r^n)}{(1-r)}$
(n=4) $a_1 = 2(3)^{4-1} = 54$
(n=5) $a_2 = 2(3)^{5-1} = 162$
$r = \dfrac{162}{54} = 3$
$S_4 = \dfrac{54(1-3^4)}{(1-3)}$
$\boxed{S_4 = 2160}$</td><td>= sum(seq(2(3)^(x-1),x,4,7))
$\Rightarrow \boxed{S_4 = 2160}$</td></tr>
</table>

matches ✓