Adv Functions - (Finite) Geometric Series / Sigma (w/calc) WS

1) $a_1 = -6$, $r = -3$, $n = 7$, $S_n = ?$

$$S_7 = \frac{-6(1-(-3)^7)}{(1-(-3))}$$

$$\boxed{S_7 = -3282}$$

2) $a_1 = 2$, $a_6 = 64$, $r = 2$
$(n = 6)$

$$S_6 = \frac{2(1-2^6)}{(1-2)}$$

$$\boxed{S_6 = 126}$$

3) $64 - 96 + 144 - \ldots$ to 8 terms

$a_1 = 64$ $r = -\frac{96}{64} = \frac{-3}{2}$ $n = 8$

$$S_8 = \frac{64(1-(-\frac{3}{2})^8)}{(1-(-3/2))}$$

$$\boxed{S_8 = -630.5 \text{ or } \frac{-1261}{2}}$$

4) $-7 - 14 - 28 - \ldots -3584$

$a_1 = -7$ $r = \frac{-14}{-7} = 2$ a_n
$n = ?$

$$\frac{-3584}{-7} = \frac{-7(2)^{n-1}}{-7}$$

$$512 = 2^{n-1}$$

$$1 + \frac{\log 512}{\log 2} = \frac{(n-1)\log 2}{\log 2} + 1$$

$$n = 10$$

$$S_{10} = \frac{-7(1-2^{10})}{(1-2)}$$

$$\boxed{S_{10} = -7161}$$

5) $a_7 = -192$ and $r = 2$

$n = 7$ $a_1 = ?$

$$-192 = a_1(-2)^6$$

$$\frac{-192}{64} = \frac{a_1 \cdot 64}{64}$$

$$a_1 = -3$$

$$S_7 = \frac{-3(1-(-2)^7)}{(1-(-2))}$$

$$\boxed{S_7 = -129}$$

6) $a_1 = \frac{1}{3}$ and $a_{10} = 6561$

$n = 10$, $r = ?$

$$\frac{6561}{1/3} = \frac{\frac{1}{3}(r)^9}{\frac{1}{3}}$$

$$19683 = r^9$$

$$\sqrt[9]{19683} = \sqrt[9]{r^9}$$

$$r = 3$$

$$S_{10} = \frac{(\frac{1}{3})(1-3^{10})}{(1-3)}$$

$$\boxed{S_{10} = 9841.\overline{3} \text{ or } \frac{29524}{3}}$$

7) $a_1 = 160$, $a_{12} = \frac{-5}{64}$, $r = -\frac{1}{2}$
$(n = 12)$

$$S_{12} = \frac{160(1-(-1/2)^{12})}{(1-(-1/2))}$$

$$\boxed{S_{12} = 106.6 \text{ or } \frac{6825}{64}}$$

8) $a_1 = -9$, $r = 2/3$, $n = 4$

$$S_4 = \frac{-9(1-(2/3)^4)}{(1-(2/3))}$$

$$\boxed{S_4 = -21.\overline{6} \text{ or } -\frac{65}{3}}$$

9) $\frac{1}{9} - \frac{1}{3} + 1 - \ldots$ to 6 terms

a_1 $r = -\frac{1}{3} \div \frac{1}{9} = -3$ $n = 6$

$$S_6 = \frac{\left(\frac{1}{9}\right)(1-(-3)^6)}{(1-(-3))}$$

$$\boxed{S_6 = -20.\overline{2} \text{ or } -\frac{182}{9}}$$

10) $162 + 54 + 18 + \ldots + \frac{2}{3}$

a_1 $r = \frac{54}{162} = \frac{1}{3}$ a_n

$n = ?$

$$\frac{\frac{2}{3}}{162} = \frac{162\left(\frac{1}{3}\right)^{n-1}}{162}$$

$$\frac{1}{243} = \left(\frac{1}{3}\right)^{n-1}$$

$$1 + \frac{\log\left(\frac{1}{243}\right)}{\log(1/3)} = \frac{(n-1)\log(1/3)}{\log(1/3)} + 1$$

$n = 6$

$$S_6 = \frac{162(1-(1/3)^6)}{(1-(1/3))}$$

$$\boxed{S_6 = 242.\overline{6} \text{ or } \frac{728}{3}}$$

11) $a_9 = -3125000$ $a_1 = -8$

$r = ?$ $n = 9$

$$\frac{-3125000}{-8} = \frac{-8(r)^8}{-8}$$

$$390625 = r^8$$

$$\sqrt[8]{390625} = \sqrt[8]{r^8}$$

$$r = 5$$

$$S_9 = \frac{-8(1-5^9)}{(1-5)}$$

$$\boxed{S_9 = -3{,}906{,}248}$$

12) $r = 1/4$, $a_6 = 36$

$n = 6$ $a_1 = ?$

$$36 = a_1(1/4)^5$$

$$1024 \cdot 36 = a_1 \cdot \frac{1}{1024} \cdot 1024$$

$$a_1 = 36864$$

$$S_6 = \frac{36864(1-(1/4)^6)}{(1-(1/4))}$$

$$\boxed{S_6 = 49140}$$

13) $a_1 = 3$, $a_n = 1029$, $r = 7$

$$\frac{1029}{3} = \frac{3(7)^{n-1}}{3}$$

$$343 = 7^{n-1}$$

$$1 + \frac{\log 343}{\log 7} = \frac{(n-1)\log 7}{\log 7} + 1$$

$n = 4$

$$S_4 = \frac{3(1-7^4)}{(1-7)}$$

$$\boxed{S_4 = 1200}$$

14) $a_3 = -36 \quad a_6 = -972, n = 10$

$a_1 = ? + r = ?$

$\div \begin{cases} -972 = \cancel{a_1} \cdot r^5 \\ -36 = \cancel{a_1} \cdot r^2 \end{cases}$

$27 = r^3$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$r = 3$

$-36 = a_1 \cdot 3^2$

$\frac{-36}{9} = \frac{a_1 \cdot 9}{9}$

$a_1 = -4$

$S_{10} = \frac{-4(1-3^{10})}{(1-3)}$

$\boxed{S_{10} = -118096}$

15) $S_n = 1530 \quad r = 2 \quad n = 8$

$a_1 = ?$

$1530 = \frac{a_1(1-2^8)}{(1-2)}$

$1530 = \frac{-255a_1}{-1}$

$\frac{1530}{255} = \frac{255a_1}{255}$

$\boxed{a_1 = 6}$

16) $S_n = 492.1875 \quad a_6 = 7.8125 \quad r = \frac{1}{2}$

$(n = 6)$

$492.1875 = \frac{a_1(1-(\frac{1}{2})^6)}{(1-(\frac{1}{2}))}$

$492.1875 = \frac{a_1 \cdot .984375}{.5}$

$\frac{492.1875}{1.96875} = \frac{1.96875a_1}{1.96875}$

$\boxed{a_1 = 250}$

17) $S_n = 1062880 \quad r = 3$

$a_1 = 4 \quad n = ?$

$1062880 = \frac{4(1-3^n)}{(1-3)}$

$1062880 = \frac{4(1-3^n)}{-2}$

$\frac{1062880}{-2} = \frac{-2(1-3^n)}{-2}$

$\underset{-1}{-531440} = \underset{-1}{1} - 3^n$

$-531441 = -3^n$

$531441 = 3^n$

$\frac{\log 531441}{\log 3} = \frac{n \log 3}{\log 3}$

$\boxed{n = 12}$

18) $S_n = 249.92 \quad r = .2 \quad n = 5, a_3 = ?$

$249.92 = \frac{a_1(1-.2^5)}{(1-.2)}$

$249.92 = \frac{a_1 \cdot .99968}{.8}$

$\frac{249.92}{1.2496} = \frac{1.2496a_1}{1.2496}$

$a_1 = 200$

$a_3 = 200(.2)^4$

$\boxed{a_3 = .32}$

19) $\sum_{n=1}^{6}\left(\frac{1}{2}\right)^n \to$ sum(seq((1/2)^x, x, 1, 6, 1)) $\Rightarrow \boxed{S_6 = \frac{63}{64}}$

20) $\sum_{n=3}^{10} 4(-3)^{n-1} \to$ sum(seq(4(-3)^(x-1), x, 3, 10, 1))

$\Rightarrow \boxed{S_8 = -59040}$

21) $\sum_{n=2}^{7} \frac{1}{4}(2)^{n-1} \to$ sum(seq((1/4)(2)^(x-1), x, 2, 7, 1))

$\Rightarrow \boxed{S_6 = 31.5 \text{ or } \frac{63}{2}}$

22) $\sum_{n=1}^{6} 5\cdot 2^{n-1}$

Method #1

$[5\cdot 2^{1-1}] + [5\cdot 2^{2-1}]$
$+ [5\cdot 2^{3-1}] + [5\cdot 2^{4-1}]$
$+ [5\cdot 2^{5-1}] + [5\cdot 2^{6-1}]$
$= 5 + 10 + 20 + 40 + 80 + 160$
$\Rightarrow \boxed{S_6 = 315}$ ✓

Method #2

sum(seq(5·2^(x-1), x, 1, 6, 1))
$\Rightarrow \boxed{S_6 = 315}$ ✓

Method #3

$S_n = \frac{a_1(1-r^n)}{(1-r)}$ $\quad a_1 = 5\cdot 2^{1-1} = 5$ $\quad n = 6 \ (6-1+1=6)$

$a_2 = 5\cdot 2^{2-1} = 10$ $\quad r = 2 \ (r = \frac{10}{5} = 2)$

$\sum_{n=1}^{6} 5\cdot 2^{n-1} \to a_n = a_1 \cdot r^{n-1}$ $\quad S_6 = \frac{5(1-2^6)}{(1-2)}$

($5 \to a_1$, $2 \to r$)

$\boxed{S_6 = 315}$ ✓